General Models for Optimum Arbitrary-Dimension FPGA Switch Box Designs

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Abstract–*An FPGA switch box is said to be hyper-universal if it is routable for all possible surrounding multi-pin net topologies satisfying the routing resource constraints. It is desirable to design hyper-universal switch boxes with the minimum number of switches. A previous work, Universal Switch Module, considered such a design problem concerning 2-pin net routings around a single FPGA switch box. However, as most nets are multi-pin nets in practice, it is imperative to study the problem that involves multi-pin nets. In this paper, we provide a new view of global routings and formulate the most general k-sided switch box design problem into an optimum k-partite graph design problem. Applying a powerful decomposition theorem of global routings, we prove that, for a fixed k, the number of switches in an optimum k-sided switch box with W terminals on each side is $O(W)$, by constructing some hyper-universal switch boxes with $O(W)$ switches. Furthermore, we obtain optimum, hyper-universal 2-sided and 3-sided switch boxes, and propose hyper-universal 4-sided switch boxes with less than $6.7W$ switches, which is very close to the lower bound $6W$ obtained for pure 2-pin net models in [5].*

1 Introduction

The well-known SRAM-based FPGA architecture [3, 5, 6] consists of an array of 2-D Logical Blocks (L-cells) separated by vertical and horizontal channels, each with W (called channel density) prefabricated wire segments (tracks) for routing, see Figure 1. Each track within a channel is assigned an integer in $\{1, \ldots, W\}$ as its track ID. There is a connection box (C-box) in the channel area between each pair of adjacent L-cells, and a switch box (S-box) at each intersection of a vertical and horizontal channels. Both C-boxes and S-boxes contain programmable switches.

When an FPGA is used to realize a specified Boolean function, the pins used to realize the Boolean function are partitioned into groups (called nets). Then the pins in each group (net) are connected together by using available wire segments and switches in

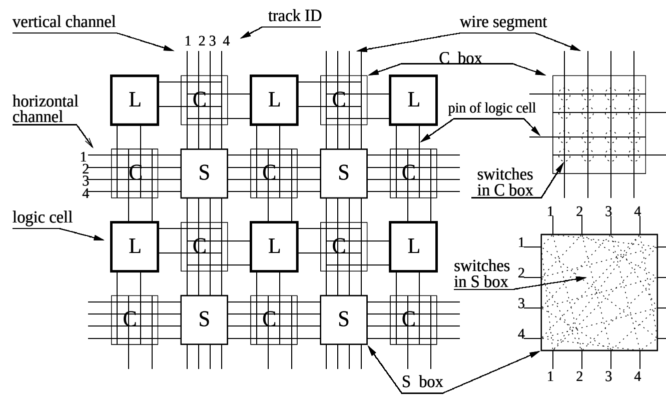

Figure 1. The architecture of a 2D-FPGA.

both C-boxes and S-boxes. This process is referred to as a routing. Conventionally, the routing process is divided into two subsequent steps, global router and detailed router, although there is no absolute need for doing routing in these two phases. The global router specifies various connection topologies for all nets, while the detailed router decides assignments of wire segments and switches used to materialize the complete routing. As the connectivity within a C-box is complete, the routability of the entire chip only depends on the structure and connectivity of the S-boxes [1, 3, 4, 5, 8, 9, 11, 12, 13, 14]. It is clearly desirable to design switch boxes with maximized routability and the minimum number of switches.

The Universal Switch Module proposed in [5] is routable for all possible global routings surrounding an S-box. However, there is a restriction that this model assumes the case of 2-pin nets only. In this paper, we propose a new view of global routings, and a powerful graph model for the most general FPGA routing problems covering multi-pin nets (including nets with ≥ 3 pins) and being adaptable to the optimum routing problems covering the entire chip.

In order to complete a detailed routing for an entire chip, a greedy routing architecture has been proposed in [11, 14]. The approach starts the detailed routing from a pre-specified S-box. Then routings of adjacent C-boxes are determined and serves as the predetermined side(s) of other neighboring (and unmapped) S-boxes. This process is repeated (propagated) until the entire chip routing is done. Depending on the propagation order (e.g., either spiral or snake-like [11, 14]), the process can be decomposed into a sequence of h-side-predetermined, k-sided S-box

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design problems for $k = 3, 4$ and $0 \leq h \leq k$. With this design scheme, we need to design an h-side-predetermined, k-sided S-box, where $2 \leq k \leq 4$ and $0 \leq h \leq k$, that can accommodate any global routing (called being hyper-universal) with the minimum number of switches. For simplicity, we call a 0-side-predetermined k-sided S-box a k-sided S-box. In this regard, the well-studied Xilinx-based S-box shown in [10] and the Universal Switch Box [5] belong to the 4-sided routing models.

In our formulated graph model G for a k-sided S-box, the track with ID j on the i-th side is denoted by a vertex $v_{i,j}$ and each switch in the S-box is represented by an edge. The global routing specified for an S-box is represented by a collection of subsets (nets) of $\{1, 2, \ldots, k\}$. A detailed routing of a net is represented by a subtree of graph G with vertices representing the pins on different sides. For example, a net of a global routing is represented by the set $\{1, 2, 3\}$, if it connects three wire segments located on sides $1, 2$, and 3, respectively. A detailed routing of this net will be represented by a tree of three vertices with its ends in $\{v_{1,j} | j = 1, \ldots, W\}$, $\{v_{2,j} | j = 1, \ldots, W\}$ and $\{v_{3,j} | j = 1, \ldots, W\}$, respectively. Therefore, the switch box design problem becomes a k-partite graph design problem, that is, to design a k-partite graph with the minimum number of edges which can realize any global routings. This flexible mathematical model can also be generalized to h-side-predetermined, k-sided S-box design problems with $k \geq 5$, and $0 \leq h \leq k$, which is useful for potential routing problems involving multiple routing dimensions.

2 Definitions and Problems

The terminology and symbols of graphs are referred to [2]. Let $G = (V(G), E(G))$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$. We denote by $|V(G)|$ and $|E(G)|$ the number of vertices and edges in G, respectively. Let $S \subset V(G)$. $G[S]$ denotes the induced subgraph of G by S. We use $v_{i_1} v_{i_2} \ldots v_{i_l}$ to denote the path with consecutive vertices $v_{i_1}, v_{i_2}, \ldots, v_{i_l}$.

Let $k \geq 2$ be an integer. A *localized k-way global routing* $GR = \{N_1, \ldots, N_l\}$ is a collection of subsets of $\{1, 2, \ldots, k\}$. For each integer i with $1 \leq i \leq k$, let d_i be the number of occurrences that i appears in GR. $d = \max\{d_1, d_2, \ldots, d_k\}$ is called the *density* of the localized global routing GR, and GR is called a *localized (k, d)-global-routing ((k, d)-GR)*. Each N_i in GR is referred to as a *net* of the localized global routing. We note that a localized global routing GR is a multiple set; two equal sets in GR represent two different nets in the routing. Note also that a net of cardinality n corresponds to an n-pin net.

A localized (k, d)-GR is called *primitive* ((k, d)-PGR) if it does not contain two unequal nets of size 1; a localized (k, d)-GR is called a *balanced (k, d)-global routing ((k, d)-BGR)* if each element of $\{1, \ldots, k\}$ appears d times. A localized global routing in practice may not be balanced but we can always make it balanced by including some singletons (1-pin nets). An *r-bounded global routing* is a global routing in which the size of each net is at most r. The case when $r = 2$ has been used as the target model in the design of universal switch modules [5].

There are two major advantages of representing k-way global routings as a collection of subsets of $\{1, \ldots, k\}$. One is that we can make use of the theory and methods in combinatorics, the other is that such a representation actually is a hypergraph and a BGR is a regular hypergraph. This hypergraph representation will help us gain valuable ideas and simplify our presentation.

Let $k \geq 2, W \geq 1$ be integers and $V_i = \{v_{i,j} | j = 1, \ldots, W\}$ for $i = 1, \ldots, k$. A k-partite graph on $(V_1, \ldots, V_k)$ is a graph with vertex set $\cup_{i=1}^{k} V_i$ and each V_i is an independent set for $i = 1, \ldots, k$. We denote a k-partite graph on $(V_1, \ldots, V_k)$ with edge set E by $((V_1, \ldots, V_k), E)$.

Let G be a k-partite graph on $(V_1, \ldots, V_k)$. A *detailed routing* of a localized (k, d)-GR $\{N_i | i = 1, \ldots, l\}$ in G is a set of mutually vertex disjoint subgraphs $\{T(N_i) | i = 1 \ldots, l\}$ of G satisfying:

(1) $T(N_i)$ is a tree of $|N_i|$ vertices, and

(2) $|V_j \cap V(T(N_i))| = 1$ if $j \in N_i$, for $i = 1, \ldots, l$.

$T(N_i)$ is called a *detailed routing of* N_i. Note that $d \leq W$.

A *hyper-universal (k, W) switch box ((k, W)-HUSB)* is a k-partite graph on $(V_1, \ldots, V_k)$ with W vertices in each part such that it contains a detailed routing for each localized (k, d)-GR with $d \leq W$. As a trivial example, the complete k-partite graph on $(V_1, \ldots, V_k)$ (in which, there is an edge joining each pair of vertices $v_{i,j}$ and v_{i_1,j_1} with $i_1 \neq i$) is a (k, W)-HUSB.

An *optimum* (k, W)-HUSB is a (k, W)-HUSB with the minimum number of edges. Clearly, the number of edges in an optimum (k, W)-HUSB is uniquely determined by k and W, which is denoted by $e(k, W)$. Therefore, our main graph design problems related to the switch box design problem are as follows.

Problem 1 *For a fixed k, determine $e(k, W)$ and find an optimum (k, W)-HUSB for any positive integer W.*

Problem 2 *For a given (k, W)-HUSB, find an efficient detailed routing algorithm.*

We note that a k-partite graph G with W vertices in each part is hyper-universal if it contains a detailed routing for any primitive and balanced localized (k, W)-GR ((k, W)-PBGR). To see this, let R be a (k, d)-GR with $d \leq W$. We first add some singletons to R to make a (k, W)-BGR, then by combining some singletons, we obtain a (k, W)-PBGR, say R'. Find a detailed routing of R' in G. A detailed routing of R can be derived from the detailed routing of R' by simply deleting the edges of those one-edge trees representing the nets of size two in R' which are obtained by combining the unequal nets of size 1. Therefore, to verify that a (k, W) S-box is hyper-universal, we only need to show that each (k, W)-PBGR is routable in the S-box.

Our approach to the S-box design problem depends on a powerful decomposition property of localized global routings.

Let GR be a (k, d)-BGR and GR' be a sub-collection of GR. If GR' is a (k, d')-BGR with $d' < d$, GR' is called a *subglobal routing* of GR. GR is said to be *minimal* if it does not contain subglobal routings. The following decomposition property was proved in [7].

Lemma 1 *For any integer $k \geq 2$, there exists an integer $f(k)$ such that any localized (k, d)-BGR GR can be decomposed into minimal, balanced, localized k-way subglobal routings with densities no more than $f(k)$. Moreover, $f(k) = k - 1$ for $k = 2, 3, 4$.*

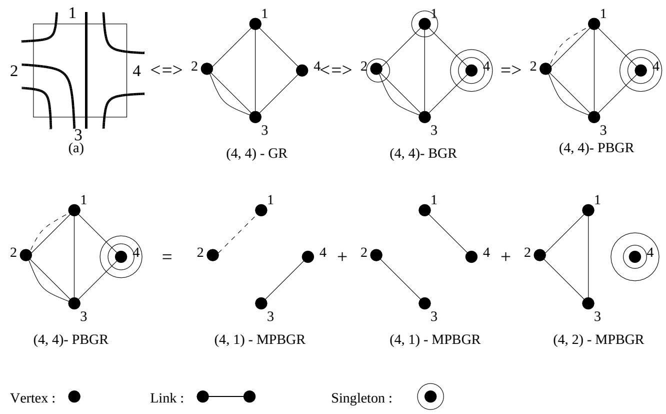


Figure 2. Examples of $(4,4)$-GR, BGR, PBGR and decomposition of the PBGR into minimal PBGRs.

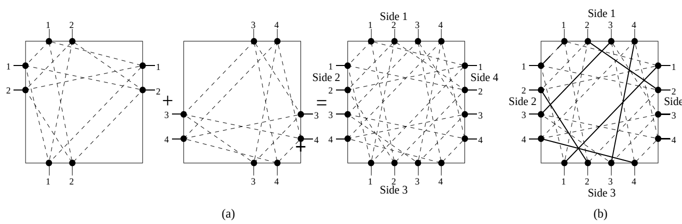


Figure 3. (a) A $(4,4)$ S-box, (b) a detailed routing.

Note that $f(k)$ is uniquely determined by k by Lemma 1, and it is equal to the maximum density of all minimal k-way BGRs.

Let GR_1 and GR_2 be two localized k-way global routings and m a positive integer. We denote the disjoint union (as a multiple set) of GR_1 and GR_2 by $GR_1 + GR_2$, and the union of m copies of GR_1 by mGR_1.

Example: Let $GR = \{\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,3\}, \{3,4\}\}$ which represents the localized global routing in Figure 2-(a). GR is a localized $(4,4)$-GR which is not balanced. $GR' = GR + \{\{1\}, \{2\}, \{4\}, \{4\}\}$ is a $(4,4)$-BGR. GR' can be transformed into a PBGR (not unique) $GR'' = \{\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,3\}, \{3,4\}, \{1,2\}, \{4\}, \{4\}\}$. This PBGR can be decomposed into the union of three minimal, localized PBGRs $GR'' = \{\{1,2\}, \{3,4\}\} + \{\{1,4\}, \{2,3\}\} + \{\{1,2\}, \{2,3\}, \{1,3\}, \{4\}, \{4\}\}$. Figure 2 shows the transformation in hypergraph notation, where the dashed link represents the 2-pin net obtained by combining two singletons, while Figure 3 shows a $(4,4)$ S-box and a detailed routing of GR'' in the box.

In the rest of the paper, a global routing refers to a localized global routing for simplicity.

3 $e(k,W) = O(W)$

For any integer $k \geq 2$, let $\{r_1, \ldots, r_t\}$ be the set consisting of all densities of minimal k-way global routings. Then $r_i \leq f(k)$, and t depends only on k, where $f(k)$ is determined by Lemma 1. Let $p(k)$ be the least common multiple of $r_1, \ldots, r_t$.

Our goal is to design all k-sided HUSBs for any fixed k with minimized number of switches. The idea is to design a few k-sided HUSBs and combine these S-boxes to obtain a (k,W)-HUSB for any W. There are many ways to do this depending on how to group the minimal balanced global routings in the decomposition given by Lemma 1. The following lemma provides one approach.

Lemma 2 *If*

$$r_1x_1 + \ldots + r_tx_t \geq tp(k) - t + 1$$

where $x_1, \ldots, x_t$ are nonnegative integers, then there are integers $0 \leq y_i \leq x_i, i = 1, \ldots, t$ such that

$$r_1y_1 + \ldots + r_ty_t = p(k)$$

Proof. By the generalized pigeon-hole principle, there is an $1 \leq i \leq t$ so that $r_ix_i \geq p(k)$. Therefore, there is an integer $y_i \leq x_i$ such that $r_iy_i = p(k)$. For any $j \neq i$, let $y_j = 0$. Then y_l is an integer with $0 \leq y_l \leq x_l$ for $l = 1, \ldots, t$ and

$$r_1y_1 + \ldots + r_ty_t = p(k).$$

This completes the proof of the lemma. ∎

By Lemma 2, we can always decompose a (k,W)-BGR into disjoint union of some $(k,p(k))$-BGRs together with at most one (k,r)-BGR, where r is determined by W and $p(k)$. Accordingly, our (k,W) S-box consists of disjoint union of some $(k,p(k))$ S-boxes and one (k,r) S-box. When k is fixed, each W corresponds to an r. But when W changes, the number of different r's corresponding to various W's is finite, since this number is less than $tp(k) - t + 1$ by Lemma 2.

Theorem 1 *For any fixed positive integer k,*

$$e(k,W) = O(W).$$

Proof. Let $r(k) = tp(k) - t + 1$. For any positive integer W, we can write $W = p(k)q + r$ where $r < r(k)$. Consider the k-partite graph $F(k,W)$ consisting of q vertex disjoint copies of complete k-partite graph $K_{(k,p(k))}$ with $p(k)$ vertices in each part, and a vertex disjoint complete k-partite graph $K_{(k,r)}$ with r vertices in each part, if $r \neq 0$. We show that $F(k,W)$ is a (k,W)-HUSB.

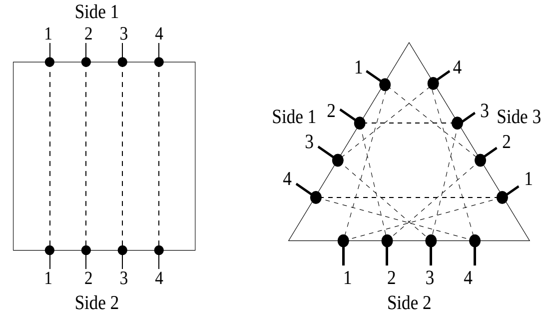
For any (k,W)-BGR GR, by Lemma 1, GR can be decomposed into a union of minimal (k,r_i)-BGR, where r_i's are defined as the above. By recursively applying Lemma 2, these minimal k-BGRs can be grouped into q $(k,p(k))$-GRs and one (k,r)-BGR if $r \neq 0$. Obviously, each $(k,p(k))$-BGR has a detailed routing in $K_{(k,p(k))}$, and if $r \neq 0$, the (k,r)-BGR has a detailed routing in $K_{(k,r)}$. This shows that $F(k,W)$ is a (k,W)-HUSB.

$$\begin{aligned} |E(F(k,W))| &= qk(k-1)p(k)^2/2 + k(k-1)r^2/2 \\ &= (qp^2(k) + r^2)k(k-1)/2 \\ &= ((W-r)p(k) + r^2)k(k-1)/2 \end{aligned}$$

It follows that $e(k,W) = O(W)$ for a fixed k. ∎

4 Optimum (k,W)-HUSBs for $k = 2,3$

Our basic method for solving the optimum (k,W)-HUSB design problem is first to give a lower bound of $e(k,W)$, then to find a k-partite graph with the number of edges equal to the lower bound and to prove that it is hyper-universal. The following theorem is obvious.



(a) (b)

Figure 4. (a) The optimum $(2,4)$-HUSB, (b) an optimum $(3,4)$-HUSB.

Theorem 2 *The graph $G(2,W)$ with vertex set $\{v_{1,j}|j = 1,\ldots,W\} \cup \{v_{2,j}|j = 1,\ldots,W\}$ and edge set $\{v_{1,i}v_{2,i}|i = 1,\ldots,W\}$ is an optimum $(2,W)$-HUSB, and $e(2,W) = W$.*

Figure 4-(a) shows an optimum $(2,4)$-HUSB.

Next we investigate the optimum $(3,W)$-HUSB design problem. Let $G(3,W)$ denote the 3-partite graph on (V_1,V_2,V_3) with edge set $\{v_{i,j}v_{h,j+(h-i)-1}|j = 1,\ldots,W; 1 \le i < h \le 3\}$ where the second index is taken modulo W. Since $j - p \neq j(\text{mod W})$ when $0 < p < W$, $v_{1,1}v_{2,1}v_{3,1}v_{1,W}v_{2,W}v_{3,W} \cdots v_{1,t}v_{2,t}v_{3,t}v_{1,t-1}v_{2,t-1}v_{3,t-1}\cdots v_{1,2}v_{2,2}v_{3,2}v_{1,1}$ is a Hamiltonian cycle of $G(3,W)$.

Theorem 3 *The graph $G(3,W)$ is an optimum $(3,W)$-HUSB, and $e(3,W) = 3W$.*

Proof. Let $G = ((V_1,V_2,V_3),E)$ be an optimum $(3,W)$-HSUB. Since there are at least W edges between any two sides by Theorem 2, $e(3,W) = |E(G)| \ge 3W$. Note that $|E(G(3,W))| = 3W$, thus, we only need to show that $G(3,W)$ is a $(3,W)$-HUSB.

Let $GR = \{N_i|i = 1,\ldots,l\}$ be any $(3,W)$-PBGR. Then it can be shown, by Lemma 1 and induction on W, that the nets in GR can be ordered and the elements in each N_i can be ordered so that $1,2,3$ appear successively in a cyclic order. For example, if $GR = \{\{1,2\},\{2,3\},\{1,3\}\}$, then GR can be ordered as $\{2,3\},\{1,2\},\{3,1\}$, (or $\{\{1,2\},\{3,1\},\{2,3\}\}$) to satisfy the required order property.

Without loss of generality we assume that the ordered sequence of N_i's is $N_1,N_2,\ldots,N_l$ and $1 \in N_1$ is the first element in the ordering. Start from $v_{1,1}$ along the Hamiltonian cycle

$$v_{1,1}v_{2,1}v_{3,1}v_{1,W}v_{2,W}v_{3,W}\cdots v_{1,t}v_{2,t}v_{3,t}$$
$$v_{1,t-1}v_{2,t-1}v_{3,t-1}\cdots v_{1,2}v_{2,2}v_{3,2}v_{1,1},$$

we successively cut a section with $|N_i|$ vertices as $T(N_i)$. Then $\{T(N_i)|i = 1,\ldots,l\}$ is a $(3,W)$-global routing of $\{N_i|i = 1,\ldots,l\}$ in $G(3,W)$, and therefore, $G(3,W)$ is a $(3,W)$-HUSB and $e(3,W) = 3W$. ∎

Remarks: *1. There are more than one optimum $(3,W)$-HUSBs for some W. For example, the disjoint union of a 6-cycle and a 3-cycle is also an optimum $(3,3)$-HUSB. But a $3W$-cycle is always an optimum $(3,W)$-HUSB by Theorem 3. An optimum $(3,4)$-design is shown in Figure 4-(b).*

2. The proof of Theorem 3 also gives an efficient algorithm to find a detailed routing of a $(3,W)$-GR in $G(3,W)$.

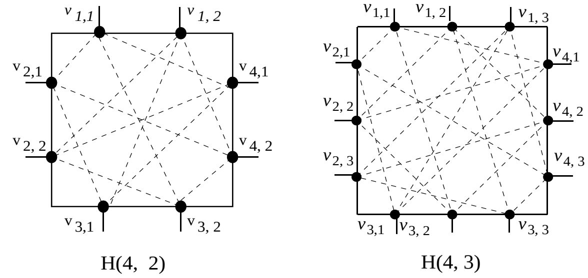


Figure 5. $H(4,2)$ and $H(4,3)$.

5 $(4,W)$-HUSBs

In this section, we first give a lower bound estimation of $e(k,W)$. Then we construct three $(4,W)$-HUSBs. The first two are connected with one being 4-regular. The third design is not connected but contains less number of edges, which provides an upper bound for $e(4,W)$.

Theorem 4

$$e(k,W) \ge \frac{k(k-1)}{2}W.$$

Proof. For each pair i,j with $1 \le i < j \le k$, let $GR(i,j)$ be a (k,W)-GR with W $\{i,j\}$'s and W $\{t\}$'s for each $t \in \{1,\ldots,k\} \setminus \{i,j\}$. A detailed routing of $GR(i,j)$ must contain at least W edges between V_i and V_j. Therefore a (k,W)-HUSB contains at least $\binom{k}{2}W$ edges. It follows that $e(k,W) \ge Wk(k-1)/2$. ∎

Next we focus on designing $(4,W)$-HUSBs. It is easy to see that a (k,W)-HUSB on $(V_1,\ldots,V_k)$ satisfies that any two parts (V_i,V_j) induces a $(2,W)$-HUSB and any three parts (V_i,V_j,V_p) induces a $(3,W)$-HUSB. Therefore, it is desirable to design a k-partite graph on $(V_1,\ldots,V_k)$ such that any two parts (V_i,V_j) induces a graph which is a perfect matching (by Theorems 2), and any three parts (V_i,V_j,V_p) induces a graph which is a cycle (by Theorems 3). We refer to such a graph as an *$MH(k,W)$-graph*.

Define $H(4,W) = ((V_1,\ldots,V_4),E)$ where $V_i = \{v_{i,j}|j = 1,\ldots,W\}$, $i = 1,\ldots,4$ and $E = \{v_{i,j}v_{i+1,j}|i = 1,\ldots,4, j = 1,\ldots,W\} \cup \{v_{1,j}v_{3,j+1}|j = 1,\ldots,W\} \cup \{v_{2,j}v_{4,j-1}|j = 1,\ldots,W\}$, in which the first index takes modulo 4 and the second index takes modulo W. See Figure 5 for $W = 2,3$. Then $H(4,W)$ is an $MH(4,W)$-graph.

We note that $|E(H(4,W))| = 6W$, which is the lower bound given by Theorem 4. It is easy to verify that $H(4,2)$ is a $(4,2)$-HUSB, and therefore it is optimum. But $H(4,3)$ is not a $(4,3)$-HUSB as it does not contain a detailed routing of the global routing $\{\{1,2\},\{1,2\},\{3,4\},\{3,4\},\{1,3\},\{2,4\}\}$. We can also verify that the optimum Universal Switch Modules given in [5] are not $(4,W)$-HUSBs.

However, we can obtain $(4,W)$-HUSBs by adding some edges to $H(4,W)$. Let $Q(4,W)$ be the graph obtained from $H(4,W)$ by adding edges $\{v_{1,j}v_{3,j}, v_{2,j}v_{4,j}|j = 1,\ldots,W\}$. We further define the graph $K(4,W)$ to be the graph obtained from $Q(4,W)$ by removing the edges $v_{2,1}v_{4,W}$ and $v_{3,1}v_{1,W}$. Figure 6 shows

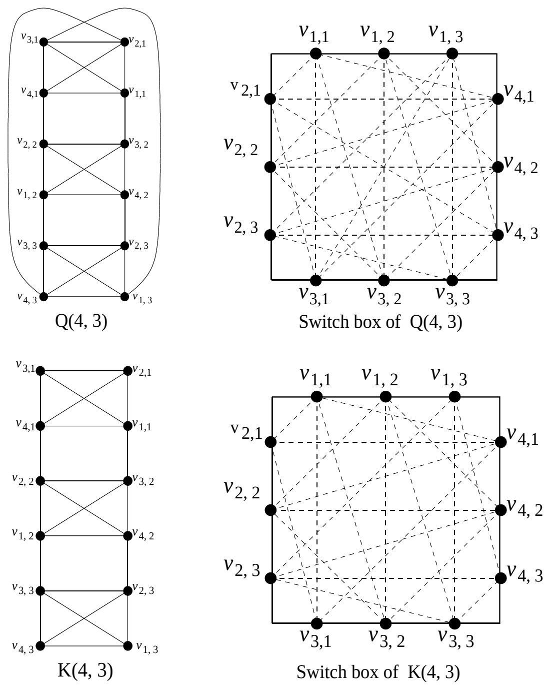


Figure 6. Two types of $(4, W)$-HUSBs.

a $Q(4,3)$ and a $K(4,3)$ in two different drawings. The hyper-universal property of the two designs is implied from the following Theorem 5.

To design and verify a $(4,W)$-HUSB, we need to find all minimal 4-way PBGRs. By Lemma 1, we can find all the 35 different minimal PBGRs, which are classified into eight equivalent classes by the permutation group S_4. In the following we list only one representative from each equivalent class. We use $\mathcal{GR}_j^i$ to denote the class of minimal $(4,i)$-PBGRs of type j. The number of elements in a class is represented by a superscript.

$\mathcal{GR}_1^1 = \{\{1,2,3,4\}\}^{(1)}$,
$\mathcal{GR}_2^1 = \{\{1,2\},\{3,4\}\}^{(3)}$,
$\mathcal{GR}_3^1 = \{\{1,2,3\},\{4\}\}^{(4)}$,
$\mathcal{GR}_1^2 = \{\{1,2,3\},\{1,2,4\},\{3,4\}\}^{(6)}$,
$\mathcal{GR}_2^2 = \{\{1,2,3\},\{1,4\},\{2\},\{3,4\}\}^{(12)}$,
$\mathcal{GR}_3^2 = \{\{1,2\},\{3,1\},\{2,3\},\{4\},\{4\}\}^{(4)}$,
$\mathcal{GR}_1^3 = \{\{1,2,3\},\{1,2,4\},\{3,4,1\},\{2,3,4\}\}^{(1)}$,
$\mathcal{GR}_2^3 = \{\{1,2,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\}\}^{(4)}$.

Let $G_1 = K_4$, $G_2 = H(4,2)$, G_3, G_4, G_5 G_6 and G_7 be as in Figure 7. Then we have $|E(G_1)| = 6$, $|E(G_2)| = 12$, $|E(G_3)| = 20$, $|E(G_4)| = 26$, $|E(G_5)| = 32$, $|E(G_6)| = 40$ and $|E(G_7)| = 46$. We note that G_3 contains vertex disjoint G_1 and G_2; G_4 contains two vertex disjoint G_2's; G_5 contains vertex disjoint G_2 and G_3; G_6 contains three vertex disjoint G_2's; and G_7 contains vertex disjoint G_3 and G_4.

Lemma 3 *G_i is a $(4,i)$-HUSB for $1 \le i \le 7$.*

Proof. It is obvious that G_1 is hyper-universal. Let GR be a $(4,i)$-PBGR. We need to show that GR is routable in G_i for $2 \le i \le 7$.

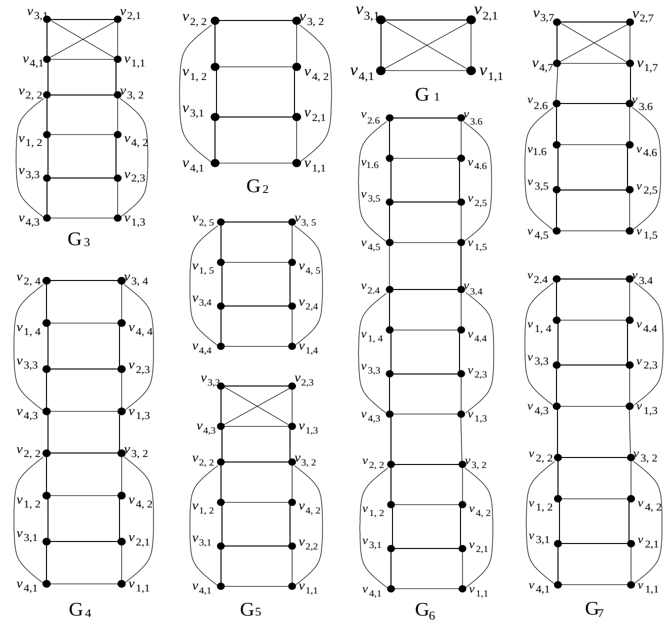


Figure 7. The elementary $(4, W)$-HUSBs.

For $i = 2$, GR is either a member in $\mathcal{GR}_1^2 \cup \mathcal{GR}_2^2 \cup \mathcal{GR}_3^2$ or a union of two members in $\mathcal{GR}_1^1 \cup \mathcal{GR}_2^1 \cup \mathcal{GR}_3^1$. It is easy to verify that G_2 is a $(4,2)$-HUSB.

Let $i = 3$. If GR is a member of $\mathcal{GR}_1^3 \cup \mathcal{GR}_2^3$, we can verify that G_3 contains a detailed routing of GR. If GR is a union of a $(4,1)$-PBGR and a $(4,2)$-PBGR, then G_3 contains a detailed routing of GR since G_3 contains vertex disjoint subgraphs G_1 and G_2.

For $i = 4$, if GR is a union of a $(4,1)$-PBGR and a minimal $(4,3)$-PBGR, then we can verify that G_4 contains a detailed routing of GR. Suppose GR is a union of two $(4,2)$-PBGRs. In this case, G_4 contains a detailed routing of GR as G_4 contains two vertex disjoint G_2's.

For $i = 5$, GR can always be decomposed into a union of a $(4,2)$-PBGR and a $(4,3)$-PBGR. It is easy to see that G_5 contains a detailed routing for GR since G_5 contains vertex disjoint G_2 and G_3.

Let $i = 6$. If GR is a union of three $(4,2)$-PBGRs, then G_6 contains a detailed routing of GR as G_6 contains three vertex disjoint G_2's. If GR is a union of two $(4,3)$-PBGRs, then we can verify that G_4 contains a detailed routing of GR.

Finally, let $i = 7$. Then GR can always be decomposed into a $(4,3)$-PBGR and a $(4,4)$-PBGR. G_7 contains a detailed routing of GR since G_7 contains vertex disjoint subgraphs G_3 and G_4.

This completes the proof of the lemma. ∎

Now we can construct a family of $(4,W)$-HUSBs according to different values of W and the elementary $(4,i)$-HUSB G_i for $i = 1,2,\ldots,7$. Define $F(W)$ as the disjoint union of graphs as follows.

$$F(W) = \begin{cases} h\ G_6\text{'s} & \text{if } W = 6h, \\ (h-1)\ G_6\text{'s and a } G_7 & \text{if } W = 6h+1, \\ h\ G_6\text{'s and a } G_2 & \text{if } W = 6h+2, \\ h\ G_6\text{'s and a } G_3 & \text{if } W = 6h+3, \\ h\ G_6\text{'s and a } G_4 & \text{if } W = 6h+4, \\ h\ G_6\text{'s and a } G_5 & \text{if } W = 6h+5. \end{cases}$$

By the definition of $G_i, i = 1, \ldots, 7$, we have that the number of edges of $F(W)$ for $W > 1$ is given by the following.

$$|E(F(W))| = \begin{cases} \frac{20}{3}W & \text{if } W = 0(mod\ 6), \\ \frac{20}{3}W - \frac{2}{3} & \text{if } W = 1(\text{mod } 6), \\ \frac{20}{3}W - \frac{4}{3} & \text{if } W = 2(\text{mod } 6), \\ \frac{20}{3}W & \text{if } W = 3(\text{mod } 6), \\ \frac{20}{3}W - \frac{2}{3} & \text{if } W = 4(\text{mod } 6), \\ \frac{20}{3}W - \frac{4}{3} & \text{if } W = 5(\text{mod } 6). \end{cases}$$

Theorem 5 *For $W > 1$, $F(W)$ is a $(4, W)$-HUSB.*

Proof. If $W = 6h+1$, then $h \geq 1$ and any $(4, 6h+1)$-PBGR GR can be decomposed into a union of $(h-1)$ $(4, 6)$-PBGRs and a $(4, 7)$-PBGR (sometimes, GR can be decomposed into h $(4, 6)$-PBGRs and a $(4, 1)$-PBGR. But this is not guaranteed). Since $F(6h+1)$ is a disjoint union of $h-1$ $(4, 6)$-HUSBs and a $(4, 7)$-HUSB, then GR is routable in $F(6h+1)$.

Let $W = 6h + i$ with $i \neq 1$. Any $(4, 6h+i)$-PBGR GR can be decomposed into h $(4, 6)$-PBGRs and a $(4, i)$-PBGR. Since $F(6h+i)$ is a disjoint union of h G_6's and a G_i if $i \neq 0$, and G_6 and G_i are $(4, 6)$-HUSB and $(4, i)$-HUSB, respectively, by Lemma 3, $F(6h+i)$ contains a detailed routing of GR.

∎

Remarks: *1. From the results of Theorems 4 and 5, we have $6W \leq e(4, W) \leq 6.\overline{6}W$, which is very close to the lower bound of $6W$ for the pure 2-pin net routings.*

2. To obtain a detailed routing $F(W)$ of a given $(4, W)$-GR, we first make it balanced and primitive, then decompose it into a disjoint union of minimal 4-way PBGRs, and then group them into some $(4, 6)$-PBGRs and a $(4, r)$-PBGR according to the construction of $F(W)$ in Theorem 5, and finally find the detailed routing for each of the subglobal routings. This process can be completed in polynomial time and therefore there is an efficient algorithm for a detailed routing in $F(W)$.

3. If we consider only 2-pin nets, then it can be easily shown that each 2-restricted 4-way BGR of density $2r$ (or $2r+1$) can be decomposed into the union of r 4-way BGRs of density 2 (or plus one 4-way BGR of density 1). Using this fact, we can similarly construct an optimum Universal Switch Box with $6W$ switches that was proposed in [5].

6 Conclusion

We have developed the first general mathematical model that covers the multi-pin net perfect routing and design problems for arbitrary-dimension FPGA switch boxes. Under the new models, the switch box design problem is formulated as an optimum k-partite graph design problem. The new models have many advantages. Firstly, it simplifies the representations of the global and detailed routings and makes it possible to use techniques in graph theory and combinatorics to attack the problem. As a result, optimum 2-way and 3-way S-boxes have been obtained. Secondly, the new model of the global routing has a powerful decomposition property which enables us to construct large (k, W)-HUSBs by combining a few number of smaller k-sided HUSBs. This construction has led to a very low cost $(4, W)$-HUSB. The decomposition property also guarantees the existence of polynomial time algorithm for detailed routing in the universal S-boxes we have designed. Thirdly, the new models enable us to generalize the k-way S-box design for $k = 2, 3$ and 4 in 2D-FPGA to general (k, W)-design problem with $k \geq 5$, which can be directly applied for the higher dimension ($\geq 3D$) switch box designs. The theory developed here can also be used to solve various switch box design problems, like h-side-predetermined, k-sided switch boxes and switch boxes for r-restricted global routings that can be applied for designing the non-homogenious greedy routing structures aiming for optimum routings covering the whole chip.

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