Moment Computations of Nonuniform Distributed Coupled RLC Trees with Applications to Estimating Crosstalk Noise

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Abstract

A novel method is proposed to compute moments of nonuniform distributed coupled RLC transmission lines. Recursive formulae of moments of coupled RC trees are extended to those of coupled RLC trees by considering both self inductances and mutual inductances. It can be observed that moment computations of distributed lines outperform those of lumped ones. The inductive crosstalk noise waveform can be accurately and efficiently estimated using the moment computation technique in conjunction with the projection-based order reduction method. Fundamental developments of the proposed approach are described in details. Experimental results demonstrate the improved accuracy of the proposed method over that of the traditional lumped methods.

1 Introduction

Modern technological trends have caused interconnect modeling to have attracted considerable attention in high-speed VLSI designs. Owing to these designs with performance considerations, increasing clock frequency, shorter rising times, higher density of wires, and using low-resistivity materials, on-chip inductance effects can no longer be ignored in interconnect models. The importance of coupling inductance effects has grown continuously since nanometer technology has emerged over the last few years. It has been observed that crosstalk noise estimations made by considering inductance effects may yield more pessimistic results than those made without considering coupling inductance effects [4]. Furthermore, the importance of coupling inductance effects has grown continuously since nanometer technology has emerged over the last few years. When estimating crosstalk noise between neighbor wires with considering inductance effects, it is advisable to consider both self inductances and mutual inductances [7, 9, 12].

This paper aims to estimating crosstalk noise for distributed coupled RLC trees with nonuniform lines. For practical designs, the nonuniform interconnects are often used to optimize the circuit performance. Many researches have focused on establishing distributed line models for simulations [1, 2, 3, 5, 6, 10, 11, 13]. However, it seems that no suitable model of nonuniform distributed lines has been developed for efficiently analyzing noises. The techniques employed are the model-order reduction methods. We use moment computation to construct a reduced-order system [8]. Every line current and voltage moments are approximated as polynomial functions, which depends on the coordinate of the line. The circuit parameters of the nonuniform lines can also be represented as polynomials by means of numerical interpolations. Under this framework, all of the coefficients of the polynomials of moments can be calculated recursively. Recursive moment computation formulas and a moment model for nonuniform distributed coupling transmission lines are also developed. The efficiency and the accuracy of moment computation of distributed lines can be shown that outperform those of lumped ones. Crosstalk-metric models for distributed coupled RLC trees are also established. A stable reduced-order model will be constructed implicitly using the recursive moment computation technique in conjunction with the projection-based model-order reduction method. Crosstalk noise estimations will be made by investigating the crosstalk noise of this reduced-order network.

The rest of this paper is organized as follows. Section 2 presents recursive formulas of moments for distributed coupled RLC-tree models. Section 3 introduces an efficient method for establishing reduced-order models with guaranteed stable poles to estimate crosstalk noise. Section 4 presents the simulation results. Finally, conclusions are made in Section 5.

2 Moments Computations

A set of coupled RLC trees includes several individual RLC trees with capacitive and inductive couplings to each other. Each RLC tree is comprised of floating resistors, self inductors, and distributed lines from the ground, and capac-

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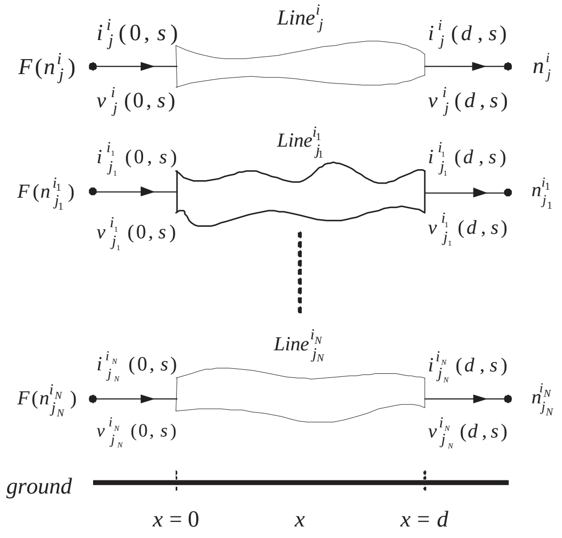

Figure 1. Coupled nonuniform transmission lines in couple RLC trees.

itors that connect nodes on the tree and to the ground. A distributed RLC-tree model excludes couplings and resistor loops. Let T^i be a tree in a set of coupled RLC trees, n^i_j be the jth node in the tree T^i, and $F(n^i_j)$ be the corresponding father node of n^i_j. A nonuniform distributed line $Line^i_j$ is indeed connected between the nodes n^i_j and $F(n^i_j)$.

Let $V^i_j(s)$ be the transfer function of the voltage at node n^i_j and $I^i_j(s)$ be that of the current that flows into n^i_j. In particular, $V^i_0(s) = V^i_s$ represents the voltage at root n^i_0, where V^i_s implies that a voltage source is connected between the root node of T_i, n^i_0, and the ground. For an aggressor tree, $V^i_s = 1$; for a victim tree, $V^i_s = 0$. Expanding $V^i_j(s)$ and $I^i_j(s)$ in power series yields $V^i_j(s) = \sum_{k=0}^{\infty} V^i_{j,k} s^k$ and $I^i_j(s) = \sum_{k=0}^{\infty} I^i_{j,k} s^k$. $V^i_{j,k}$ is called the kth-order *voltage moment* of $V^i_j(s)$ and $I^i_{j,k}$ is called the kth-order *current moment* of $I^i_j(s)$. This section aims to compute moments $V^i_{j,k}$ and $I^i_{j,k}$ for each node n^i_j of a given order k. We first focus on the moment model of the coupled distributed line system and then insert the moment model into coupled RLC trees.

2.1 Moments of Coupled RLC Lines

Consider the N-conductor coupled distributed line system, shown in Fig. 1, where $x = 0$ and $x = d$ represent the near end and the far end of the lines, respectively. Let $r^i_j(x)$, $l^i_j(x)$, and $c^i_j(x)$ be the per-unit-length resistance, inductance, and capacitance of the nonuniform line $Line^i_j$. The per-unit-length $g^i_j(x)$ for each x is assumed to be neglected. $c^{i,i_1}_{j,j_1}(x)$ and $m^{i,i_1}_{j,j_1}(x)$ represent the per-unit-length coupling capacitance and the per-unit-length mutual inductance between $Line^i_j$ and $Line^{i_1}_{j_1}$, respectively. cc^i_j and mm^i_j represent the set of coupling capacitances and mutual inductances connected to $Line^i_j$. In general, the coupling effect, especially inductive coupling, is not restricted to arising only between two closest neighbors. Consequently, the proposed method addresses the general circumstances in which each set cc^i_j and mm^i_j may involve many coupling capacitances and mutual inductances.

Applying the Laplace transformation, the telegrapher's equation of $Line^i_j$ can be rewritten as follows:

$$\begin{aligned} \frac{\partial v^i_j(x,s)}{\partial x} &= -\big(r^i_j(x) + s l^i_j(x)\big) i^i_j(x,s) \\ &\quad - \sum_{mm^i_j} \big(s m^{i,i_1}_{j,j_1}(x) i^{i_1}_{j_1}(x,s)\big) \end{aligned} \tag{1}$$

$$\begin{aligned} \frac{\partial i^i_j(x,s)}{\partial x} &= -s c^i_{jT}(x) v^i_j(x,s) \\ &\quad + \sum_{cc^i_j} \big(s c^{i,i_1}_{j,j_1}(x) v^{i_1}_{j_1}(x,s)\big) \\ &= i^i_{cj}(x,s), \end{aligned} \tag{2}$$

where $c^i_{jT}(x) = c^i_j(x) + \sum_{cc^i_j} c^{i,i_1}_{j,j_1}(x)$. $v^i_j(x,s)$ and $i^i_j(x,s)$ are the voltage and current at x in the frequency domain. The following lemma states the recursive moment formulae of the kth-order moments of $v^i_j(x,s)$ and $i^i_j(x,s)$.

Lemma 1 (Recursive moment formulae): Let $v^i_{j,k}(x)$, $i^i_{j,k}(x)$, and $i^i_{cj,k}(x)$ be the kth-order moment of $v^i_j(x,s)$, $i^i_j(x,s)$, and $i^i_{cj}(x,s)$, respectively. If $k = 0$, each capacitance behaves as an open circuit, then the zeroth-order moments $i^i_{cj,0}(x) = i^i_{j,0}(d) = 0$ and $v^i_{j,0}(x) = v^i_{j,0}(0) = V^i_s$. For $k > 0$,

$$i^i_{cj,k}(z) = c^i_{jT}(z) v^i_{j,k-1}(z) - \sum_{cc^i_j} \big(c^{i,i_1}_{j,j_1}(z) v^{i_1}_{j_1,k-1}(z)\big), \tag{3}$$

$$i^i_{j,k}(x) = i^i_{j,k}(d) + \int_x^d i^i_{cj,k}(z) dz, \tag{4}$$

$$\begin{aligned} v^i_{j,k}(x) &= v^i_{j,k}(0) - \int_0^x r^i_j(z) i^i_{cj,k}(z) dz - R^i_j(x) i^i_{j,k}(x) \\ &\quad - \int_0^x l^i_j(z) i^i_{cj,k-1}(z) dz - L^i_j(x) i^i_{j,k-1}(x) \\ &\quad - \sum_{mm^i_j} \Big(\int_0^x m^{i,i_1}_{j,j_1}(z) i^{i_1}_{cj_1,k-1}(z) dz \\ &\quad + M^{i,i_1}_{j,j_1}(x) i^{i_1}_{j_1,k-1}(x) \Big), \end{aligned} \tag{5}$$

where $R^i_j(x) = \int_0^x r^i_j(z) dz$, $L^i_j(x) = \int_0^x l^i_j(z) dz$, and $M^{i,i_1}_{j,j_1}(x) = \int_0^x m^{i,i_1}_{j,j_1}(z) dz$ represent total resistance, inductance, and mutual inductance of $Line^i_j$ at the length x.

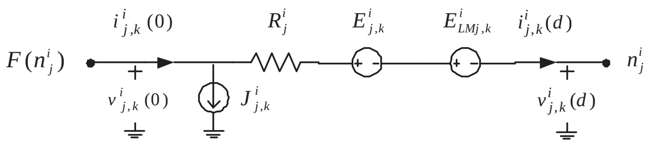

Figure 2. Moment model of couple RLC lines.

To reduce the computational cost of the integrations in (4) and (5), the line current and voltage moments, $i^{\,i}_{cj,k}(x)$ and $v^i_{j,k}(x)$, are approximated as polynomials [12],

$$i^i_{cj,k}(x) = \sum_{n=0}^{m_k} \alpha^i_{j,k,n} x^n,\ v^i_{j,k}(x) = \sum_{n=0}^{p_k} \beta^i_{j,k,n} x^n. \qquad (6)$$

Also, all circuit parameters $r^i_j(x)$, $l^i_j(x)$, $c^i_j(x)$, $c^{i,i_1}_{j,j_1}(x)$, and $m^{i,i_1}_{j,j_1}(x)$ are approximated as q-degree polynomials, in which the coefficients can be obtained by using interpolation techniques. Thus each integral item in (5) can be calculated simply by polynomial multiplications as follows:

$$\begin{aligned}
\int_0^x r^i_j(z) I^i_{cj,k}(z)dz &= \sum_{n=0}^{q+m_k+2} a^i_{j,k,n} x^n, \\
R^i_j(x) I^i_{j,k}(x) &= \sum_{n=0}^{q+m_k+2} b^i_{j,k,n} x^n, \\
\int_0^x l^i_j(z) I^i_{cj,k-1}(z)dz &= \sum_{n=0}^{q+m_{k-1}+2} c^i_{j,k-1,n} x^n, \\
L^i_j(x) I^i_{j,k-1}(x) &= \sum_{n=0}^{q+m_{k-1}+2} d^i_{j,k-1,n} x^n, \\
\int_0^x m^{i,i_1}_{j,j_1}(z) I^{i_1}_{cj_1,k-1}(z)dz &= \sum_{n=0}^{q+m_{k-1}+2} e^i_{j,k-1,n} x^n, \\
M^{i,i_1}_{j,j_1}(x) I^{i_1}_{j_1,k-1}(x) &= \sum_{n=0}^{q+m_{k-1}+2} f^i_{j,k-1,n} x^n.
\end{aligned}$$

It is worthy of mentioning that all these coefficients can be calculated recursively by using the method similar to the recursive moment computation algorithm in [8]. The following lemma shows the relationship of the degrees of the coefficients α's and β's.

Lemma 2: If $k = 0$, the zeroth-order polynomials $i^i_{cj,0}(x) = 0$ and $v^i_{j,0}(x) = V^i_s$ imply that $\alpha^i_{j,0,0} = 0$ and $\beta^i_{j,0,0} = V^i_s$. Thus $m_0 = p_0 = 0$. For $k > 0$, it can be derived that $m_k = q+(k-1)(2q+2)$ and $p_k = k(2q+2)$.

Once coefficients α's and β's are obtained, the moment model of coupled RLC lines, shown in Fig. 2, can be established. From (4), by setting $x = 0$, we obtain

$$i^i_{j,k}(0) = i^i_{j,k}(d) + J^i_{j,k}, \qquad (7)$$

where $J^i_{j,k} = \int_0^d I_{cj,k}(z)dz = \sum_{n=0}^{m_k} \frac{\alpha^i_{j,k,n}}{n+1} d^{n+1}$ represents the total capacitive current of $Line^i_j$. Similarly, (5) can be rewritten as below:

$$v^i_{j,k}(d) = V^i_{j,k}(0) - \sum_{n=0}^{q+m_k+2} b^i_{j,k,n} d^n - E^i_{LMj,k} - E^i_{j,k}, \qquad (8)$$

where

$$E^i_{LMj,k} = \sum_{n=0}^{q+m_{k-1}+2} (d^i_{j,k-1,n} + \sum_{mm^i_j} f^i_{j,k-1,n}) d^n,$$

$$E^i_{j,k} = \sum_{n=0}^{q+m_k+2} a^i_{j,k,n} d^n + \sum_{n=0}^{q+m_{k-1}+2} (c^i_{j,k-1,n} + \sum_{mm^i_j} e^i_{j,k-1,n}) d^n.$$

$E^i_{j,k}$ and $E^i_{LMj,k}$ represent the voltage drops from voltage moment $v^i_{j,k}(0)$, resulted from the kth-order and the $(k-1)$st-order capacitive current moment flowing through resistance and inductance of $Line^i_j$, respectively. Note that for coupled RC lines, $E^i_{LMj,k} = 0$.

2.2 Inserting Coupled Lines Into Coupled RLC Trees

Suppose that $line(n^i_j) = 1$ means a line connected between nodes n^i_j and $F(n^i_j)$; otherwise, $line(n^i_j) = 0$. Let R^i_j and L^i_j be the resistance and the inductance, respectively, connected between n^i_j and $F(n^i_j)$; $C^i_{j,0}$ is the capacitance connected between n^i_j and the ground. C^{i,i_1}_{j,j_1} represents the coupling capacitance between n^i_j and $n^{i_1}_{j_1}$; M^{i,i_1}_{j,j_1} is the mutual inductance between L^i_j and $L^{i_1}_{j_1}$. CC^i_j represents the set of coupling capacitances connected to n^i_j; MM^i_j is the set of mutual inductances coupled to L^i_j. $S(n^i_j)$ represents the set of the son nodes of n^i_j.

In order to incorporate coupled distributed lines into moment computation algorithm, the recursive formulae are required to be updated. The kth-order current moments can be written as

$$I^i_{j,k} = I^i_{Cj,k} + \sum_{n^i_y \in S(n^i_j)} I^i_{y,k}, \qquad (9)$$

where $I^i_{y,k} = I^i_{y,k} + line(n^i_y) \cdot J^i_{y,k}$. Each current moment can be calculated from the leaves to the root of tree T^i. Considering the relationships between voltage moments $V^i_{j,k}$ and $V^i_{F(j),k}$ gives

$$\begin{aligned}
V^i_{j,k} \;=\; & V^i_{F(j),k} - R^i_j I^i_{j,k} - L^i_j I^i_{j,k-1} - \sum_{MM^i_j} M^{i,i_1}_{j,j_1} I^{i_1}_{j_1,k-1} \\
& - line(n^i_j) \cdot (E^i_{j,k} + E^i_{LMj,k}). \qquad (10)
\end{aligned}$$

The computational complexity of the recursive formulae of the distributed model is $\mathcal{O}(nk^2)$, where n is the number of nodes in the trees. However, The computational complexity of the recursive formulae of the lumped model is equal to $\mathcal{O}(mk)$, where m is the number of nodes in the tree [8]. Therefore, under general situations where $m > nk$, in terms of computational complexity, the distributed model seems to be superior to the lumped one. Simulation results in Section 4 will demonstrate this fact.

3 Estimating Crosstalk Noise with the Stable-Pole Model

The crosstalk waveform model can be approximated by $\widehat{v}(t) = k_1 e^{p_1 t} + k_2 e^{p_2 t} + \cdots + k_q e^{p_q t}$, where all of k_i and p_i for $1 \le i \le q$ are poles and residues of the q-pole reduced-order model $\widehat{V}(s)$. The guaranteed stable poles can be yielded by solving the roots of $|s\widehat{M} + \widehat{N}| = 0$, where matrices $\widehat{M}$ and $\widehat{N}$ are generated by the congruence transformation of the MNA matrices M and N [8, 12]. In recent study [13], distributed lines, which are infinite-order systems, are modeled as finite-order macro models by the integrated-congruence transform. Thus the MNA formula can be constructed as follows:

$$\Bigg(s\underbrace{\begin{bmatrix} \widehat{M}_d & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & L \end{bmatrix}}_{M} + \underbrace{\begin{bmatrix} \widehat{N}_d & -A_d & 0 \\ A_d^T & G & A_l \\ 0 & -A_l^T & R \end{bmatrix}}_{N}\Bigg) \underbrace{\begin{bmatrix} \widehat{X}_d(s) \\ V_n(s) \\ I_L(s) \end{bmatrix}}_{X(s)} = \underbrace{\begin{bmatrix} 0 \\ A_s \\ 0 \end{bmatrix}}_{b} V_s(s), \tag{11}$$

where $(s\widehat{M}_d + \widehat{N}_d)\widehat{X}_d(s) = A_d V_n(s)$ are the state equations of the reduced-order model of distributed lines; V_n and I_L represent the node voltage vector and R-L branch current vector; the matrices R, L, G, and C include lumped resistors, inductors, conductors, and capacitors; and A_d, A_l, and A_s are incidence matrices.

Let vector $X_k = [\widehat{X}_{d,k}\ V_{n,k}\ I_{L,k}]^T$ be the kth-order moment of $X(s)$ about $s = 0$. If $Q = [X_0\ X_1 \cdots X_{q-1}]$ is used as the congruence transform matrix, then the MNA matrices of the reduced-order model are obtained: $\widehat{M} = Q^T M Q$ and $\widehat{N} = Q^T N Q$ [10]. Thus, the kth-row and the lth-column entry of $\widehat{N}$ and $\widehat{M}$ become $\widehat{n}_{k,l} = X_{k-1}^T N X_{l-1}$ and $\widehat{m}_{k,l} = X_{k-1} M X_{l-1}$, respectively. The entries in $\widehat{M}$ and $\widehat{N}$ are subtly related, summarized in the following proposition.

Proposition 1: The entries of $\widehat{M}$ and $\widehat{N}$ are have the following subtle relationships:

1. $\widehat{m}_{ij} = -X_{i-1}^T N X_j = -\widehat{n}_{i,j+1}$.
2. $\widehat{m}_{ij} = X_{j-1}^T M X_{i-1} = -X_{j-1}^T N X_i = -\widehat{n}_{j,i+1}$.

Therefore, except $\widehat{m}_{kk}$, all entries in $\widehat{M}$ can be determined directly from $\widehat{N}$. The remaining task is to calculate each entry in $\widehat{N}$ and the entry $\widehat{m}_{kk}$. Since the recursive moment formula implies $NX_{k+1} = -MX_k$,

$$\begin{aligned} \widehat{n}_{k,l} &= -X_{k-1}^T M X_{l-2} \\ &= -(\widehat{X}_{d,k-1}^T \widehat{M}_d \widehat{X}_{d,l-2} + V_{n,k-1}^T C V_{n,l-2} \\ &\quad + I_{L,k-1}^T L I_{L,l-2}), \end{aligned} \tag{12}$$

where $V_{n,k-1}^T C V_{n,l-2}$ and $I_{L,k-1}^T L I_{L,l-2}$, related to lumped circuits, can be calculated by the technique in [8]. For $\widehat{X}_{d,k-1}^T \widehat{M}_d \widehat{X}_{d,l-2}$, Proposition 2 shows the results.

Proposition 2: The term $\widehat{X}_{d,k-1}^T \widehat{M}_d \widehat{X}_{d,l-2}$ can be calculated by evaluating the contribution of each distributed line. The contributions of a line $Line_j^i$ are

$$\int_0^d v_{j,k-1}^i(z) i_{Cj,l-1}^i(z) dz = \sum_{n=0}^{p_{k-1}+m_{l-1}} \frac{g_{j,n}^i}{n+1} d^{n+1},$$

and

$$\int_0^d i_{j,k-1}^i(z) \Big(L_j^i(z) i_{j,l-3}^i(z) + \sum_{mm_j^i} \big(M_{j,j_1}^{i,i_1}(z) i_{j_1,l-3}^{i_1}(z) \big) \Big) dz = \sum_{n=0}^{q+m_{k-1}+m_{l-3}+3} \frac{h_{j,n}^i}{n+1} d^{n+1}.$$

The entries in the matrix $\widehat{N}$ can be simplified further, as stated in the following proposition.

Proposition 3: The entries in the first column and the first row of matrices $\widehat{N}$ have the following relationships:

1. $\widehat{n}_{11} = 0$;
2. $\widehat{n}_{i1}$ $(i > 1)$, denoted as $I_{1,i-1}^a$, is equal to the $(i-1)$st-order moment of the current entering node n_1^a in the aggressor tree T^a;
3. $\widehat{n}_{1i} = -\widehat{n}_{i1}$.

4 Simulation Results

Three coupling circuits, presented in Fig. 3, are studied to estimate crosstalk noise and verify the accuracy of the proposed method. The squares represent the roots of the trees and the circles represent the leaves of the trees. In all circuits, the line parameters are resistance: $3.50 - 8.53 \cdot 10^{-3}x + 1.05 \cdot 10^{-4}x^2\ \Omega/mm$; capacitance: $0.55 + 3.31 \cdot 10^{-3}x - 1.32 \cdot 10^{-5}x^2\ pF/mm$; self inductance: $0.27 - 6.60 \cdot 10^{-4}x + 8.09 \cdot 10^{-6}x^2\ nH/mm$; coupling capacitance: $0.47 + 6.61 \cdot 10^{-3}x - 2.63 \cdot 10^{-5}x^2\ pF/mm$; mutual inductance: $0.12 + 6.60 \cdot 10^{-4}x - 8.09 \cdot 10^{-6}x^2\ nH/mm$.

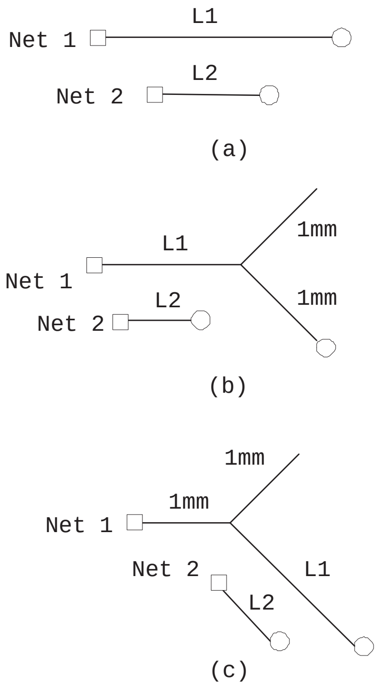

Figure 3. Three types of coupled RLC trees: (a) two lines (b) tree 1 (c) tree 2. The lengths of the coupling lines of net 1 belong to the set $L1 = \{1, 2, 3, 4, 5\}(mm)$ and those of net 2 are also in the set $L2 = \{1, 2, 3, 4, 5\}(mm)$; the latter are never longer than the former.

The load on each line is 50fF. Peak values of noise is considered for circuits of different cases that involve different circuit topologies, line lengths, coupling locations, effective driver impedances, and rising times. In the circuit of Fig. 3, the lengths of the coupling lines of net 1 belong to the set $L1 = \{1, 2, 3, 4, 5\}(mm)$ and those of net 2 are also in the set $L2 = \{1, 2, 3, 4, 5\}(mm)$; the latter are never longer than the former. Other branches in Fig. 3(b)(c) are all 1 mm long. The coupling locations between nets 1 and 2 are different and are shifted 1mm from alignment at the near end of net 1 toward the far end of net 1. Additionally, four effective driver impedance pairs: 3 Ω - 3 Ω, 3 Ω - 30 Ω, 30 Ω - 3Ω, and 30 Ω - 30 Ω, connected to the near ends of the two nets, are studied. In each case, nets 1 and 2 are independently excited. The voltage source connected to the aggressor net is a ramp function with two rising times, 0.02ns and 0.2ns, and with a normalized unit magnitude. Thus a total of 1640 cases are used to examine the accuracy of the proposed method.

The conventional one-pole model (1P) and two-pole model (2P) [12] and the new method with the three-pole model

Table 1. Comparisons of the absolute and relative errors of the crosstalk peak values.

Abs. (%)	1P	2P	S3P	S4P	S5P	S6P
Max.	104.21	82.30	20.16	25.53	19.77	12.81
Avg.	16.27	12.66	4.57	3.16	2.23	1.26
Min.	0.04	0.18	0.02	0	0	0
Rel. (%)	1P	2P	S3P	S4P	S5P	S6P
Max.	312.44	182.24	57.85	73.48	63.57	49.89
Avg.	63.52	43.08	20.33	15.93	11.34	6.38
Min.	0.12	1.49	0.03	0.02	0	0

Table 2. Computational times of moments and relative errors of model S6P generated by lumped couple RLC-tree models with different number of sections per 1mm long.

Section No.	2	3	4	5	6
Time (sec.)	32.34	56.35	63.46	81.03	95.22
Error (%)	8.80	7.68	7.18	6.97	6.80
Section No.	10	20	30	40	50
Time (sec.)	167.06	325.23	478.96	657.48	902.13
Error (%)	6.63	6.46	6.42	6.40	6.39

(S3P), the four-pole model (S4P), ..., and the six-pole model (S6P) are comparatively investigated. Table 1 summarizes the absolute and relative errors of the crosstalk peak values, determined by comparison with HSPICE (the wire resistance, capacitance, and inductance distributed per 20μm). Of the 1640 cases, model 1P has unstable poles in 40 cases and model 2P is unstable in 15 cases. To compare the efficiency and the accuracy between distributed lines and lumped ones, Table 2 summarizes the computational times and the relative errors of model S6P. Simulation results yield the following observations:

1. The models generated by the proposed method outperform the conventional 1P and 2P models. Thus these conventional models are no longer appropriate for coupled RLC trees. Increasing the order of the reduced-order models allows the proposed models perform more accurately.

2. From the viewpoints of the absolute errors in Table 1, model S3P, whose average errors are smaller than 10 %, seems acceptable for estimating cross-talk noise. However, the relative errors imply that model S3P seems not accurate as expected. Model S6P is recommended to balance computational efficiency and estimation performance. As an illustration, Fig. 4 shows the crosstalk waveforms of Spice, S3P, S4P, and S6P

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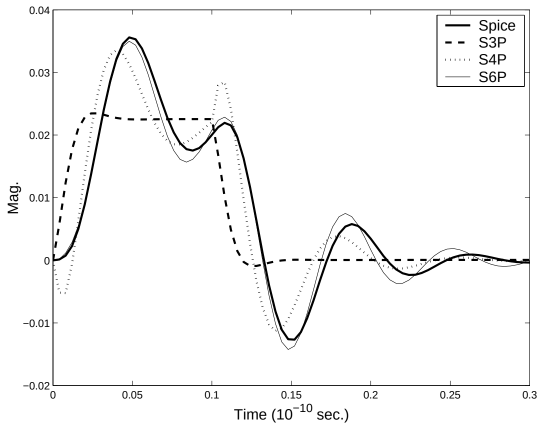

Figure 4. Crosstalk waveforms of Spice, S2P, S3P, and S6P models.

models with two coupled lines $L1 = L2 = 1mm$. Obviously, the waveform of model S6P is much more similar to that of SPICE than that of model S3P.

3. The computational time of moments and the relative errors of Model S6P yielded by the proposed method are 29.56 seconds and 6.38 $\%$, respectively. Table 2 displays that it must cost more than 902.13 seconds to obtain the same relative error by using lumped coupled RLC lines. Obviously, the efficiency and the accuracy of distributed models outperform those of lumped models.

5 Conclusion

This paper proposed new recursive formulas for computing moments of distributed coupled RLC trees with nonuniform lines. A method for estimating inductive crosstalk noise for coupled RLC-tree models is also described. The peak value of the crosstalk waveform can be estimated efficiently by using the proposed moment computations of the RLC-tree models to approximate the waveform of coupling noise. Simulation results also demonstrate the improved efficiency and accuracy of the proposed method.

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