Introduction of a New Selection Parameter in Genetic Algorithm for Constrained Reliability Design Problems

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Abstract. In this article we propose to introduce a new selection parameter in Genetic Algorithms (GAs) for a class of constrained reliability design problems. Our work demonstrates two major points. The first one is that the populations are quickly included in the space of the feasible solutions for a sufficiently large selection of parameter value. The second one is that the value of the selection parameter controls the exploration strategy of the feasible space. These two properties illustrate that an adapted choice of the selection parameter value allows to improve the performance of GA. Furthermore, our numerical examples tend to show that, with an adapted choice of the selection parameter, these GAs are in practice more efficient than previously proposed GAs for this class of problems.

1 Introduction

Over the last few decades, due to the number of occurring accidents, safety has taken an increasingly significant role. Intense research has been carried out to obtain optimal safety system at minimal cost. In this paper we are interesting in a class of Reliability Design Problems (RDP) described in [7], [10]. These problems deal with the system structure and component choice optimization in order to obtain the best compromise between system reliability and cost. These problems are most frequently NP-hard. If conventional optimization tools, for example, integer programming, Lagrangian method, dynamic programming [9] are often ineffective, the use of GAs, in the RDP, provide some promising results. But in practice GAs are extremely complex to use because of the parameter tuning and the existence of many different ways to apply the mutation crossing-over and selection process [13]. The analysis of the given solutions usually requires an important experience in GAs practice. At the end, the choice of a particular GA is usually based on heuristic approaches, empirical results and GAs expert judgements. As far as we know, few theoretical works are related to the study of the performance of GA. Most of them [1], [2], [11], [12], [14] propose a modelisation based on Markov theory in order to describe its stationary properties. None of these studies prove that the population of GA converge towards

K. Deb et al. (Eds.): GECCO 2004, LNCS 3103, pp. 90–101, 2004.

a population only constituated by solutions which are global optimum even if the number of generations tends to infinity. By extending some classical results on Simulated Annealing [4], [5] to GA, Cerf [6] proves this convergence of a new GA with three parameters allowing the control of a specified cooling-schedule. However, the application of this new GA might vary a lot from one problem to another and seems to be very complicated to implement and to use. As far as we know this GA has never been implemented. Nevertheless, practical-oriented-studies seem to confirm that the time-dependent assumption for the parameters in Evolutionary Algorithms (EAs) is well fitted [18]. The choice and the design of optimal time-dependent parameters function are at least as difficult to find the best static as parameters. But in practical studies, they have proven their efficiency at least, with EA. The idea of letting the algorithm adjust its own parameters for free is indeed appealing [18]. The realistic formulation of RDP leads to generally constrained or highly constrained problems. However the application of GA on highly constrained optimization problem could fail to give the right solution, i.e. the optimal or nearly optimal solution [15]. Different approaches are used in order to improve the GA performance like repair operators and specific penalty functions. Because the repairs operators method is not well-fitted to our problems [10], we decide to adopt the penalty approach which seems to give promising results. There are three different ways to apply penalty methods in the selection process [17] : the *static method*, the *dynamic method* [16] which modifies the penalty coefficient along the evolution according to a user-defined-schedule and the *adaptive method* which gleans information from the population in order to update the value of penalty functions. All these methods focus on the tuning of the penalty function to access more rapidly to the feasible space. These approaches are particularly interesting to find a first feasible solution when the proportion of feasible solutions is very small (approximately less than one pourcent). When the proportion of feasible solutions increases, upper than 10%, the control of the selection force, allows to direct the population very quickly in the feasible space. In fact, if the size of the feasible space is large, the exploration strategy of the feasible space has an important impact on the efficiency of the GA. In this article we propose to demonstrate that the selection parameter give the possibility to choose different exploration strategies of the feasible space. Consequently an adapted choice of this selection parameter allows to improve the efficiency of the GA. Our main goal is to propose both an efficient and practical optimization tool for constrained problems particularly encountered in RDP. In the first section, a modelisation of the GA proposed in [9] for a practical RDP based on Markov theory allows to characterize all the different steps of a GA and, particularly, the crossing-over. It is interesting to underline that, as far as we know, this is the first time that this crossing-over process has been modelled. This crossing-over is very difficult to model because of the addition of a random number of children. In the second section, we explain why and how we modify GA. In the third section, we apply the theory developed in [3] [6] on optimization problems with particular space structures in order to demonstrate the efficiency of the proposed GAs. Finally in the last section, we present some asymptotical results of modified GA performance, illustrated by a numerical comparison.

2 Mathematical Model of the Classical GA

A general optimization problem is the maximization of an objective function $f : E \to [1; +\infty[$ subject to several constraints. The solution space E can be partitioned into the feasible solution space E_R and the unfeasible solution space $\bar{E_R}$. Let X_n be the population at the n-th iteration. A realization of the random population X_n is a sequence of m solutions $x = (x_1, x_2, \ldots, x_m) \in E^m$ where $x_i \in E$, $i = 1, \ldots, m$ is a vector of L binary variables (0 or 1) and denoted $x_i = (x_i^1, x_i^2, \ldots, x_i^L)$. Let $P(X_{n+1} \mid X_n)$ be the one-step probability transition matrix. A one-step transition from X_n to X_{n+1} is divided in three distinct phases: the mutation process, the crossing-over process and the selection process. Hence, the evaluation of the probability matrix $P(X_{n+1} \mid X_n)$ requires the characterization of each process.

2.1 Modeling of the Mutation Process

A mutation is a switch of a bit from 0 to 1 (or 1 to 0) with a given probability p_m (with $0 < p_m < 1$). Let U_n be the random population obtained after the mutation process of the population X_n, and u be the associated realization vector where $u = (u_1, u_2, \ldots, u_m) \in E^m$. The mathematical expression of the probability to obtain u_i after carrying out the mutation process on x_i is determined in [14]. Let d be the Hamming distance. The expression of the mutation process transition matrix α from X_n to U_n is, $\forall (x, u) \in E^m \times E^m$:

$$\alpha(x, u) = \prod_{i=1}^{m} \underbrace{p_m^{d(x_1,u_1)}}_{\text{mutation}} \times \underbrace{(1 - p_m)^{L-d(x_1,u_1)}}_{\text{no mutation}}$$

2.2 Modeling of the Crossing-over Process

The considered crossing-over process recombines the pair of consecutive chromosomes with probability p_c in order to generate new chromosomes namely children by applying the classical one-point crossing-over rule described in page 6 of [13]. Let $CO_K(u_1, u_2)$ be the two children generated by the one point crossing-over process applied on two chromosomes (u_1, u_2), given the cut site K. One can remark that not all the couples generate two children. If a couple (u_{2i-1}, u_{2i}) does not generate, we then introduce two "virtual" chromosomes called "empty chromosome" denoted $v_\emptyset$. Note that, if the solution space dimension m is odd, the last chromosome u_m remains alone and, according to this new notation, generates an empty chromosome $v_\emptyset$. Let $E_\emptyset = E \cup \{v_\emptyset\}$ be the extended solution space. Let β_{E^2} be the probability to obtain (v_1, v_2) after carrying out one point crossing-over process on (u_1, u_2) :

$$\begin{aligned} \beta_{E^2}: \quad & E^2 \times (E_\emptyset)^2 \rightarrow \mathbb{R}^+ \\ & ((u_1,u_2),(v_1,v_2)) \mapsto \underbrace{\frac{p_c}{L-1}\sum_{k=1}^{L-1} \delta_2(CO_K(u_1,u_2);(v_1,v_2))}_{\text{expression1}} \\ & \qquad + \underbrace{(1-p_c).\delta_2((v_\emptyset,v_\emptyset);(v_1,v_2))}_{\text{expression2}} \end{aligned} \tag{1}$$

where δ_j is a comparison function of 2 vectors of the same length j defined by, $\forall j \leq m$,

$$\begin{aligned} \delta_j: \quad & E_\emptyset^j \times E_\emptyset^j \rightarrow \mathbb{R}^+ \\ & ((u_1,\ldots,u_j);(v_1,\ldots,v_j)) \mapsto \begin{cases} 1 \text{ if } u_i = v_i, \ \forall i \leq j \\ 0 \text{ else} \end{cases} \end{aligned}$$

The two expressions in the right side of equation (1) correspond to :

- expression 1: the probability that the couple of parents (u_1,u_2) generates the children (v_1,v_2), in respect with the cutting site K.
- expression 2: the probability that no reproduction occurs with the couple (u_1,u_2).

In order to completely characterize the crossing-over process, we only have to consider all the opportunities for the one-point crossing-over. Let V_n be the random population obtained after the crossing-over process of the population, and v be the associated realization vector. The crossing-over process is characterized by the transition matrix β from U_n to V_n defined by, $\forall (u,v) \in E^m \times (E_\emptyset)^m$:

$$\beta(u,v) = \prod_{1\leq i\leq p} \beta_{E^2}\big((u_{2i-1},u_{2i}),(v_{2i-1},v_{2i})\big) \qquad \text{If } m = 2p$$

$$\beta(u,v) = \delta_1(v_m,v_\emptyset)\prod_{1\leq i\leq p} \beta_{E^2}\big((u_{2i-1},u_{2i}),(v_{2i-1},v_{2i})\big) \qquad \text{If } m = 2p+1$$

2.3 Modeling of the Selection Process

The selection process consists in creating a new population X_{n+1} by applying the proportional selection described in [14] on the population composed of U_n and V_n. Because of the introduction of $v_\emptyset$, we have to extend the objective function f to $f_\emptyset$ defined by:

$$\begin{aligned} f_\emptyset : E_\emptyset &\rightarrow \mathbb{R}^+ \\ s &\mapsto \begin{cases} f(s) & \text{if } s \in E \\ 0 & \text{if } s = v_\emptyset \end{cases} \end{aligned}$$

Let $z = (z_1, z_2, \ldots, z_{2m}) \in E^m \times E_\emptyset^m$ be a vector of the global population $U_n \times V_n$ and F be the probability to select the individual z_k from the population $z = (z_1, z_2, \ldots, z_{2m})$:

$$\begin{aligned} F : \{1,\ldots,2m\} \times E^m \times E_\emptyset^m &\rightarrow \mathbb{R}^+ \\ (k, z_1 \ldots, z_{2m}) &\mapsto \frac{f_\emptyset(z_k)}{\sum_{j=1}^{2m} f_\emptyset(z_j)} \end{aligned} \tag{2}$$

It can be demonstrated that the selection process is described by the transition matrix γ :

$$\forall(y,z) \in E^m \times \big(E^m \times E_\emptyset^m\big),\ \gamma(z,y) = \prod_{r=1}^{m} \Big[\sum_{k:z_k=y_r} F(k,z) \Big] \tag{3}$$

2.4 Characterisation of the Classical GA

Finally, with the evaluation of the the probabilities α, β γ and, considering all possible paths between the population X_n and X_{n+1}, we can define the one step transition probability of the classical GA:

$$P(X_{n+1} = y \mid X_n = x) = \sum_{(u,v)\in E^m \times E_\emptyset^m} \alpha(x,u)\beta(u,v)\gamma\big((u,v),y\big)$$

With the proposed modeling, we can easily demonstrate that this classical GA does not converge as previously demonstrated in [6], [12].

3 Introduction of a Selective Force Parameter

The purpose of this section is to propose a modified GA that quickly directs its research towards a part of the space which may contain the optimal solution. The modified GA that we propose is based on the results on simulated annealing [5] and on GAs [6]. In order to control the search, we have decided to introduce a new parameter ℓ which can be seen as the force to select the solutions in the population $U_n \times V_n$. Thus in the same way that we try to optimize the function f, we are going to obtain the maximization of the following function :

$$\begin{aligned} f_{\emptyset,\ell} : E_\emptyset &\to \mathbb{R}^+ \\ s &\mapsto \begin{cases} \exp(\ln(f(s))\ell) & \text{if } s \in E \\ 0 & \text{if } s = v_\emptyset \end{cases} \end{aligned}$$

The entire selection process remains the same one excepted for the mathematical expression of the objective function. Thus we deduce the mathematical expression of F_ℓ and γ_ℓ which are respectively the proportional selection probability and the matrix transition associated to ℓ by remplacing respectively in the expression 2 $f_\emptyset$ by $f_{\emptyset,\ell}$ and by remplacing in the expression 3 $F(k,z)$ by $F_\ell(k,z)$.

The mutation and reproduction transition matrices do not change. We denote by X_n^ℓ the population associated to the modified GA with the selection parameter ℓ. The transition probability of the chain $(X_n^\ell)_{n\geq 0}$ is given by: $\forall \ell \geq 1$,

$$P(X_{n+1}^\ell = x \mid X_n^\ell = y) = \sum_{(u,v)\in E^m \times E_\emptyset^m} \alpha(x,u)\beta(u,v)\gamma_\ell\big((u,v),y\big)$$

Let us define the set $D(y)$ of all the couples (u,v) which join any point x to y by:

$$\begin{aligned} D(y) &= \{(u,v) \in E^m \times E_\emptyset^m,\ \alpha(x,u)\beta(u,v)\gamma_\ell\big((u,v),y\big) > 0\} \\ &= \{(u,v) \in E^m \times E_\emptyset^m,\ \beta(u,v) > 0 \text{ and } [(u,v)] \supseteq [y]\} \end{aligned}$$

Where $[y]$ represents the chromosomes of the vector y. With this notation, we can have:

$$P(X^\ell_{n+1} = x \mid X^\ell_n = y) = \sum_{(u,v)\in D(y)} \alpha(x,u)\beta(u,v)\gamma_\ell\big((u,v),y\big) \tag{4}$$

The introduction of the parameter ℓ does not change the properties of the Markov chain and consequently $(X^\ell_n)_{n\geq 0}$ is an homogenous, irreducible, aperiodic Markov chain that admits an unique invariant probability measure μ_ℓ.

If we increase the value of the selection parameter, the associated modified GA is more selective. In order to study the asymptotical behavior of the Modified GA, we propose to focus on the two extreme cases $\ell = 1$ and $\ell = +\infty$.

If $\ell = 1$, the modified GA is simply the Classical GA. Because of its proportional selection process, the major drawback of this algorithm, for our specific RDP, is that it conserves in the current population many solutions which do not satisfy the constraints. This is particularly important when the feasible space solution is very small compared to the whole solution space. Consequently this GA consumes a lot of time in treating unfeasible solutions.

When ℓ tends to infinity, the respective transition probabilities of the mutation and crossing over process do not change. On the other hand, the selection process tends towards a transition probability $\gamma_{+\infty}$ which can be proven to be:

$$\gamma_{+\infty}(z,y) = \lim_{\ell\to+\infty} P\big(X^\ell_{n+1} = y \mid (U^\ell_n, V^\ell_n) = z\big) = \prod_{r=1}^{m} \frac{1_{\hat{z}}(y_r) z(y_r)}{|\hat{z}|} \tag{5}$$

Where, $\forall z = (z_1, \ldots, z_{2m}) \in E^m \times E^m_\emptyset$, we have:

- $\hat{z} = \{z_k : 1 \leq k \leq 2m,\ z_k \in E,\ f(z_k) = \hat{f}(z)\}$ with $\hat{f}(z) = \max_{\{1\leq k\leq 2m,\ z_k\in E\}} f(z_k)$
- $\forall i \in E,\ z(i) = |\{1 \leq k \leq 2m,\ z_k = i\}|$
- $1_{\hat{z}}(y_k)$ is zero if y_k belongs to $\hat{z}$ and zero if not.

This GA has a well adapted property for our specific reliability design problem. The proof of equation (5) is not given here but this one can be interpreted as follow : if there exists in the current population $(U_n \times V_n)$ at least one feasible solution then all the population X_{n+1} after the selection process is necessarily in the feasible space solution. Nevertheless, its exploration strategy of the feasible space could be not adapted for some problems because of the elimination after the mutation and crossing-over process, of all the feasible solutions which are not an optimal solution of the population. Nevertheless, in the next section we are going to prove that it is possible to determine a group of GA appropriated for eliminating unfeasible solutions where each of these GAs has a different strategy of the feasible space exploration.

4 Performance of the Modified GAs for a Class of RDP

We propose in the first subsection 4.1 to characterize the asymptotical behavior of the modified GA. This approach allows us to deduce how the selection parameter direct the strategy of the whole exploration space. Consequently, we clearly show, that a sufficiently large value of the selection parameter ℓ allows to eliminate very quickly the unfeasible solutions. And at the same time, the value of the selection parameter could determine a feasible space exploration strategy which favours the diversity or, on the contrary, which favours the intensification towards the best solution of the feasible space. Finally, in the subsection 4.2, by underlining the correspondance between the length of the population and the value of the selection force parameter ℓ, we demonstrate that the modified GA will be able to concentrate very quickly its search in the restricted area of the feasible solutions.

4.1 Asymptotical Behavior

In this subsection, a study of the behavior of modified GAs probability transition when ℓ is large is presented. By transposing the reasoning proposed in [6] p. 55–57 to our modelisation, we can demonstrate that:

$$P(X_{n+1}^{\ell} = y \mid X_n^{\ell} = x) \underset{\ell \to +\infty}{\propto} \exp(-V^*(y)\ \ell) \tag{6}$$

Where

$$V^*(y) = \min_{(u,v)\in D(y)} \sum_{k=1}^{m} \big[\ln(\hat{f}(u,v)) - \ln(f(y_k))\big] \tag{7}$$

This quantity is not null only for populations verifying $V^*(y) = 0$. Consequently, when ℓ becomes very large, all the population y which has a strictly positive value of $V^*(y)$ tends to be eliminated. And we can also demonstrate:

Lemma 1.

$$\forall y \in E^m,\ \ V^*(y) = 0 \ \Leftrightarrow\ y \in S$$

Finally, with the proposition (1.15) p. 19 in [5] we can easily demonstrate that $\lim\limits_{\ell\to\infty} \mu_\ell(y) > 0 \Leftrightarrow\ y \in S$ and so, the modified GA may concentrates its exploration towards the population which are in S.

However, we prefer a GA which concentrates quickly its search towards the populations composed of the best feasible solutions. The equation 6 illustrates, that when the value of ℓ increases, the search is directed towards the populations y with smallest value of $V^*(y)$. Let G be the set of the populations which contain at least one unfeasible and one feasible solution. We assume that a static penalty method is used. Because of the large value of the static penalty, it can be shown that:

$$\forall y_1 \in G, \forall y_e \notin G,\ \ V^*(y_1) >> V^*(y_2)$$

Consequently, for any sufficiently large value of ℓ, GA eliminates the population contained in G. In the next subsection, with any sufficiently large value of m, we prove that this property allows to eliminate the unfeasible solutions. Thus, for any sufficiently large values of ℓ and m, GA concentrates its search towards the feasible solutions. Furthermore, in RDP, for any feasible solution, the corresponding objective function f is equal to the reliability of the system plus one. The values of f are in $[1, 2]$. Thus, for any population composed only of feasible solutions, the value of $V^*(y)$ is small. Consequently, because of the equation 6, if the value of ℓ is not too large it seems to be possible to keep in the population of GA a variety of feasible solutions. However, if we increase the value of ℓ, the exploration of the feasible solutions space becomes more selective. Finally, in tuning different large values of ℓ, we obtain GAs that eliminate unfeasible solutions. And the exploration strategy for such GAs in the feasible space favors more or less the population diversity.

4.2 New GAs That Concentrate Quickly Their Search towards the Space of the Feasible Solutions

The aim of this subsection is to propose a GA which eliminate the unfeasible solutions with a probability close to one after only one iteration (Theorem 1). For that, we observe that the set of the population which contains some unfeasible solutions can be partioned into two sets. The first set contains at least one unfeasible and one feasible solution.

As we have already said in the last subsection, for a sufficiently large value of ℓ, the population which belongs to the first group can be eliminated. In order to eliminate the population which belongs to the second group composed only by the unfeasible solutions, we suggest to use the fact that the feasible solution space $\bar{E_R}$ is very small compared to the whole space solution E. The uniform distribution of the initial population X_0, permit to obtain:

$$P(X_0 \in \bar{E_R}^m) = \left(\tfrac{|\bar{E_R}|}{|E|}\right)^m > 0$$

which will tend to zero when the value of m tends to infinity. Hence we have $\forall \varepsilon > 0$, $\exists m \in \mathbb{N}$, $P(X_0 \in \bar{E_R}^m) < \frac{\varepsilon}{2}$. This last property is exactly the same for the population U_0 because of the conservation of the uniform distribution after the mutation process. So we have:

$$P(U_0 = u) = \sum_{v \in E_\emptyset^m} P\big((U_0, V_0) = (u, v)\big) = \frac{1}{|E|^m} \underset{m \to +\infty}{\longrightarrow} 0 \tag{8}$$

By a similar demonstration in [6] p. 55, we can demonstrate that if the population (u, v) contains at least one feasible solution and the population y is composed only with unfeasible solutions, we have:

$$\lim_{\ell \to \infty} P\big(X_1^\ell = y \mid (U_0^\ell, V_0^\ell) = (u, v)\big) = 0$$

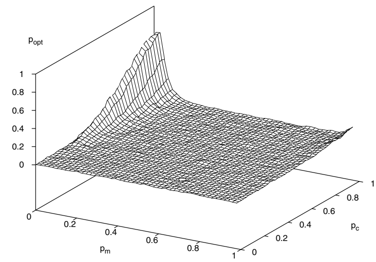
By using equations (4) and (8), $\forall y \in \bar{E_R}^m$, $\forall \ell \geq 1$ we can easily demonstrate:

$$P(X_1^\ell = y) \leq \sum_{\substack{(u,v) \in E^m \setminus \bar{E_R}^m \times E_\emptyset^m \\ [y] \subseteq [u,v]}} \frac{P\big(X_1^\ell = y \mid (U_0^\ell, V_0^\ell) = (u,v)\big)}{|E|^m} + P(X_0 \in \bar{E_R}^m)$$

Hence, we have proven that the probability $P(X_1^\ell = y)$ tends to zero when ℓ and m tend to $+\infty$ for all populations that just contain the unfeasible solutions. We have to remind that we have proven at the beginning of this subsection, that this result is also true for populations containing at least one unfeasible and one feasible solutions for any m. Finally, we have proven that :

Theorem 1. $\forall\ \varepsilon > 0\ \exists m_\varepsilon > 0\ ,\ \ \exists\ \ell_\varepsilon > 0 : \forall\ y \in E^m,\ \{y\} \cap \bar{E_R}^m \neq \emptyset,$
$\forall\ m \geq m_\varepsilon,\ \ \forall \ell \geq \ell_\varepsilon$

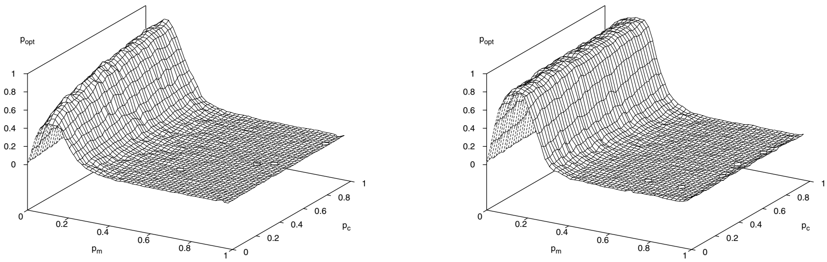
$$P(X_1^\ell = y) \leq \varepsilon\ \square$$



(a) Classical Selective GA

4.3 Numerical Results

In order to illustrate the effectiveness of our new GAs for constrained RDP problem, we will deal with a numerical example described in [8]. This example is a problem for distributing redundant units in order to maximize the reliability of a system subject to three non-linear and separable constraints. The choice of this example is motivated by the fact that this problem belongs to the well-know class of RDP [7] where the reliability expression are available [7]. We propose to apply three different GAs for solving this example the Modified GA with a large value of the selection force parameter ($\ell = 150$), the Modified GA with a very large value of the selection force parameter ($\ell = 1000$) and the "Classical Selective GA" which is very similar to a GA frequently used to solve these problems and described in [8]. These two GAs have exactly the same selection process. This selection process uses a static penalty method which sets the fitness value of any



(b) Modified GA with ℓ=150 (c) Modified GA with ℓ=1000

Fig. 1. Probabilities to obtain the optimal solution after 20 generations when p_m and p_c vary from 0 to 1 - parameters: population size = 100 - number of trials = 1000

unfeasible solution equal to -9999. The process of selecting the chromosomes in order to form the population of the next generation is completely deterministic. It selects m chromosomes from the current population and the set of offspring together in decreasing order of the fitness values. The mutation and crossing over process are described respectively in the Subsection 2.1 and in the Subsection 2.2. The Fig.1(b), 1(c) and 1(a) show the respective probabilities for the Modified GA with $\ell = 150$, the Modified GA with $\ell = 1000$, and the "Classical Selective GA" to obtain the optimal solution in a given number of generations when the mutation and crossing-over probabilities vary from 0 to 1. To obtain these results, the experiments are carried out by applying this three GAs with a population size of 100 chromosomes, the number of generations is 20 for given p_m, p_c and the same codage as proposed in [8]. The probability to obtain the optimal solution has been estimated by computing the average of successful experiments on 1000 trials. The feasible space is composed of 262144 solutions. The proportion of the feasible solutions is 20.8%. Thus, it is strongly possible that in a population of 100 chromosomes, there is at least one feasible solution. The solutions are coded with 18 bits and more than 90% of the feasible solutions have between six and nine bits coded by one. Thus the mutation and crossing-over process apply on feasible solutions seem to be able to generate new feasible solutions. Consequently, to explore the space of feasible solutions it seems to be more appropriated to have a population constituated only by feasible solutions after the selection process. However, the population of the "Classical Selective GA" after the selection process are composed by many unfeasible solutions, because of the deterministic aspect of the selection process. On the contrary, for any sufficiently large value of the selection parameter, Modified GAs allows to obtain a population after the selection process only composed of feasible solutions. Thus, Modified GA seems to be more appropriated for this numerical problem than the

"Classical Selective GA". This is illustrated by the results which demonstrate for sufficiently large value of ℓ, that modified GA obtain a better performance than the "Classical Selective GA" for any value of p_c and any value of p_m in the interval $[0, 0.2]$. Thus, the application of the modified GA is more easier because of the determination of the appropriated values of p_m and p_c. It is interesting to underline that the superiority of these new GAs should be more important for larger problems. We also remark, when the value of ℓ increases, that the results of Modified GA are improved. Obviously when ℓ is equal to 150 all the unfeasible solutions in the population of Modified GA are quickly eliminated. Thus the increasing of the selection parameter value to 1000, seems to have no consequence on the elimination of the unfeasible solutions. On the contrary this increasing seems to have an important impact on the strategy of the feasible space exploration. The selective strategy seems to be the most appropriated strategy of the feasible space exploration.

5 Conclusion

In this paper, we have proven that an adapted choice of the selection parameter ℓ allows to obtain the elimination of the unfeasible solutions with a very high probability, near to one, and allows to obtain an efficient strategy of the feasible space exploration. This last aspect, when the size of the feasible space is large, makes possible to decrease the time of computation. Consequently, for RDP, where the proportion of feasible solutions is not too small, an adapted choice of the selection parameter seems to lead GA more quickly to the convergence. In addition the selection parameter is used exactly in the same way that the mutation and the crossing-over probability. Consequently, for the users of GA, it is very easy to change the strategy of the space exploration by tuning different values of the selection parameter. Nevertheless, it is difficult to determine exactly the value of the selection parameter. Thus, it turns out to be clear that further theoretical and experimental studies could improve the performance of GA for this kind of problems. Finally, it is also interesting to extend this work to optimization problems which have a solution space which contains several disjoint feasible subspaces of solutions.

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