Evolving Golomb Rulers

Jorge Tavares[1], Francisco B. Pereira[1,2], and Ernesto Costa[1]

[1] Centre for Informatics and Systems of the University of Coimbra,
Polo II - Pinhal de Marrocos, 3030 Coimbra, Portugal
[2] Instituto Superior de Engenharia de Coimbra,
Quinta da Nora, 3030 Coimbra, Portugal
{jast, xico, ernesto}@dei.uc.pt

1 Introduction

A Golomb Ruler is defined as a ruler that has marks unevenly spaced at integer locations in such a way that the distance between any two marks is unique. Unlike usual rulers, they have the ability to measure more discrete measures than the number of marks they possess. Although the definition of a Golomb Ruler does not place any restriction on the length of the ruler, researchers are usually interested in rulers with minimum length. An Optimal Golomb Ruler (OGR) is defined as the shortest length ruler for a given number of marks [1].

2 Evolution with Marks

We propose a straightforward chromosome representation that allow us to evolve both the number and position of marks for a given ruler with a predetermined maximum length L. An individual is codified as a binary string with length L, where each bit is associated with an integer position in the ruler. If a given gene has the value 1 then it indicates that there is a mark in the corresponding position. To generate descendants we selected variants of standard genetic operators usually applied to binary representations: two-point crossover and shift mutation. Shift mutation acts in the following way: it randomly selects a mark and shifts it left or right with equal probability. To determine how many marks are shifted in a given individual, a value is uniformly selected from the integer interval $[1, S]$, where S is a small constant. To evaluate an individual we consider two criteria: ruler length and legality of the solution. The equation used to assign fitness to an individual x is the following:

$$fitness(x) = \begin{cases} \text{- number of repeated measurements} & \text{, if } x \text{ is illegal} \\ \text{number of marks} \times \text{L} + \text{(L - M)} & \text{, if } x \text{ is legal} \end{cases} \quad (1)$$

Where L is the maximum length of the ruler and M is the length of the ruler encoded in the individual.A correction and insertion procedure is performed during the evaluation of an individual. The aim is twofold: to fix invalid rulers and to see if it is possible to add marks to legal solutions. The line of action is straightforward: a mark is randomly removed if, when checking the segments

K. Deb et al. (Eds.): GECCO 2004, LNCS 3103, pp. 416–417, 2004.

measured by the ruler, a duplicate measurement is found. A constant C gives the maximum number of rectifications allowed for a given solution. If a valid ruler is obtained at the end of the correction, then the second stage can proceed: an insertion operator tries to add marks in every possible position ensuring that the obtained ruler is still valid.

3 Experimental Results

To evaluate our approach (Marks-EC), we used it to seek for good rulers with lengths between 90 and 200 (given the known OGRs we will be looking for rulers between 12 and 16 marks). In table 1 we summarize the results achieved by Marks-EC and compare them with previous evolutionary approaches: Solidays EC [2] and RK, RK heuristic [3]. We also present the optimal length for each one of these rulers.The overall results attained by Marks-EC clearly outperform previous approaches. It was able to find the OGR for all tested instances with the single exception for 15 marks. Nevertheless, for OGR-15 the solution discovered is just 2:65% higher than the optimal value, whilst the previous best value found by an evolutionary approach is 7:28% higher. A detailed analysis of the results can be found at [4].

Table 1. Overview of the results.

Instances	Optimal	Soliday's EC	RK	RK Heuristic	Marks-EC
OGR-12	85	103	91	85	85
OGR-13	106	124	111	106	106
OGR-14	127	168	134	131	127
OGR-15	151	206	160	162	155
OGR-16	177	238	193	180	177

References

1. Golomb, S.: How to Number a Graph. In: Graph Theory and Computing. Academic Press (1972) 23–37
2. Soliday, S., Homaifar, A., G., L.: Genetic algorithm approach to the search for golomb rulers. In: Proceedings of the Sixth International Conference on Genetic Algorithms (ICGA-95), Morgan Kaufmann (1995) 528–535
3. Pereira, F.B., Tavares, J., Costa, E.: Golomb rulers: The advantage of evolution. In: Proceedings of the 11th Portuguese Conference on Artificial Intelligence, Workshop on Artificial Life and Evolutionary Algorithms (ALEA), EPIA'03. (2003) 27–33
4. Tavares, J., Pereira, F.B., Costa, E.: Understanding the role of insertion and correction in the evolution of golomb rulers. In: Proceedings of the Congress on Evolutionary Computation (CEC 2004). (2004)