

# Empirical Study of Population Diversity in Permutation-Based Genetic Algorithm

Kenny Q. Zhu and Ziwei Liu

Department of Computer Science, National University of Singapore, Singapore 119260  
kzhu@comp.nus.edu.sg, lziwei@hotmail.com

**Abstract.** This paper presents an empirical study of population diversity measure and adaptive control of diversity in the context of a permutation-based algorithm for Traveling Salesman Problems and Vehicle Routing Problems.

Maintenance of diversity adaptive control is one of the most fundamental issues of Genetic Algorithm (GA). We are concerned in this paper with GA in which individual chromosomes are integer encoded, and the crossover operations are *permutation-based* [3]. Here, all individuals have the same set of distinct alleles (integers) in all generations. The different permutations of the individuals decode into different fitness values. Permutation-based GA is used to solve Traveling Salesman Problem (TSP) [2], Vehicle Routing Problem (VRP) [1], and many other problems. Permutation-based crossover operator *Partially Matched Crossover (PMX)* [3] is used in this research, with an application rate of  $p_c$ . We use a *sequence insertion* mutation, which is defined as relocating a subsequence from one position of the chromosome to another position. Probability of mutation is  $p_m$ .

We define the *phenotypes (ptype)* diversity measure as the number of unique fitness values in the population, divided by the size of the population.

With a fixed, 100% random initial population, and PMX crossover only, the basic algorithm was run 10 times with  $p_c = 0.7$  and  $p_m = 0$ . In each generation, ptype is recorded and plotted in Fig. 1, which demonstrates the natural convergence of the ptype diversity without any mutation. We further ran the algorithm to 201 generations for 50 times, and plot the rankings of accumulated diversity over 201 generations against the rankings of the mean fitness at the 201st generation, and this is shown in Fig. 2. One can observe that the more diverse the populations are in a run, the better the search quality. In other words, there is some positive correlation between diversity and search quality.

Fig. 3 and Fig. 4 illustrate the effect of genetic operations on the ptype diversity. One can observe from these plots that both crossover and mutation promote diversity, with increasing application rate. The correlation between diversity and search quality motivates us to maintain diversity at some desirable level so that more promising regimes in the search space can be explored. And the effect of crossover and mutation suggests that one can control the diversity by adaptively calibrating the crossover/mutation rates against the changing population

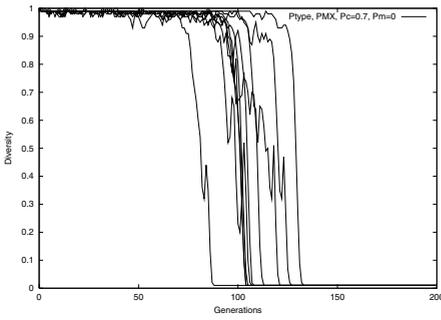


Fig. 1. Ptype vs generations

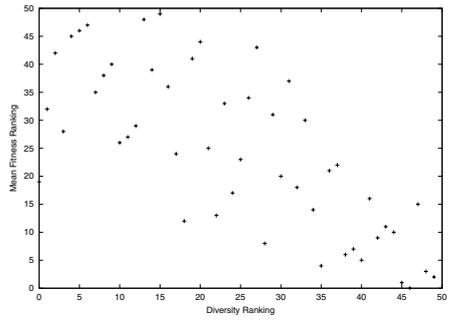


Fig. 2. Ptype rankings vs mean fitness rankings

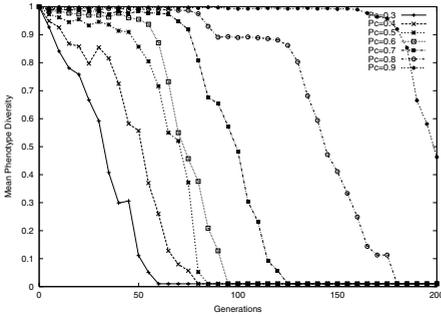


Fig. 3. Effect of PMX on Ptype

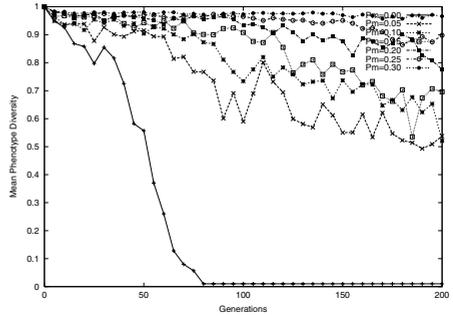


Fig. 4. Effect of mutation on Ptype

diversity. We apply the following simple adaptive function.

$$p' = \max(p_{min}, \min(p_{max}, p(1 + \frac{\xi(d_t - d)}{d}))), \tag{1}$$

where  $p$  is the current rate  $p_c$  or  $p_m$ ,  $p'$  is the new rate in the next generation,  $d$  is the diversity of current population,  $d_t$  is the target diversity,  $\xi$  is the control sensitivity, and  $p_{min}$ ,  $p_{max}$  are constants. Our preliminary experiments on various VRP benchmark problems indicated that adaptive control outperforms the search quality of fixed parameter GA by 1-5%.

## References

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