A Subpopulation Stability Based Evolutionary Technique for Multimodal Optimization

Rodica Ioana Lung

Babeş Bolyai University of Cluj Napoca, Department of Computer Science
srodica@cs.ubbcluj.ro

Abstract. A new evolutionary technique for multimodal optimization called *Roaming technique (RT)* is proposed. Multiple optima are detected using subpopulations evolving in isolation. A stability measure is defined for subpopulations by which they are characterized as stable or unstable. Stable subpopulations are considered to contain local optima. An external population called *the archive* is used to store the optima detected. After a number of generations the archive contains all local optima. Experimental results prove the efficiency of the algorithm.

1 Introduction

Most real world problems require the detection not only of one global optimum but also of other global or local optima. In some cases the local optima may be almost as good as the global one, or they may provide a human decision maker with a better insight into the nature of the design space.

Several evolutionary approaches to the multimodal optimization have been made. Among them we mention: fitness sharing [1], crowding [2], deterministic crowding [3], Multinational GA [4], Forking GA [5] and adaptive elitist-population based GA [6].

2 Roaming Optimization

A new evolutionary technique called *roaming* is proposed. Roaming technique identifies the local optima using isolated subpopulations and stores them into an external population called the *archive*.

Subpopulations are characterized as stable or unstable using a stability measure. The stability measure was intuitively arrived at. Unstable subpopulations evolve in isolation until they become stable. There is no interaction between subpopulation at any stage, therefore no coevolution takes place within roaming technique. Stable subpopulations are supposed to contain local optima.

Subpopulation stability does not imply the concentration of its individuals in a certain region of the search space. This presents an advantage over other subpopulation methods.

The number of subpopulations is a parameter of the algorithm and it is not related to the expected number of local optima. This confers flexibility and robustness to the roaming search mechanism.

After a number of generations the archive contains all local optima.

3 Roaming Technique

Consider the optimization problem:

$$\begin{cases} \text{maximize } eval(x), \\ x \in S, \end{cases}$$

where S is the solution space and $eval(x)$ is the fitness value of individual x.

Let N be the number of subpopulations. At each generation t the population $P(t)$ is composed of N subpopulations $P_i(t)$, $i = 1, \ldots, N$.

We may define an order relation on $P(t)$.

Definition 1. *We say that individual x* is better then *y, and we write $x \succ y$, if and only if the condition*

$$eval(x) \geq eval(y),$$

holds.

3.1 Stability Measure

A *stability measure* is introduced for determining whether a subpopulation has located a potential optimum.

By evolving subpopulation $P_i(t)$ a new subpopulation $P_i^{'}(t)$ having the same size as $P_i(t)$ is obtained.

Let x_i^* be the best individual in the parent subpopulation $P_i(t)$. We define an operator B as the set of individuals in the offspring in subpopulation $P_i^{'}(t)$ that are better then x_i^*:

$$B : P(t) \longrightarrow \mathcal{P}(P(t))$$

$$B(x_i^*) = \{x \in P_i^{'}(t) \mid x \succ x_i^*\}.$$

Using the cardinality of the set B the stability measure $SM(P_i(t))$ of subpopulation $P_i(t)$ may be defined.

Definition 2. Stability measure *of the subpopulation $P_i(t)$ is the number $SM(P_i(t))$ defined as*

$$SM(P_i(t)) = 1 - \frac{card\, B(x_i^*)}{card\, P_i(t)},$$

where x_i^* is the best individual in $P_i(t)$ and card A represents cardinality of the set A.

Proposition 3. Stability measure of a subpopulation P has the following properties:

(i) $0 \leq SM(P) \leq 1$;
(ii) If $SM(P) = 1$ then x^* is a potential local optimum;

where x^* is the best individual in P.

Proof. It is obvious using stability measure definition.

Definition 4. *A subpopulation P is called* σ-stable *if the condition*

$$SM(P) \geq \sigma \tag{1}$$

holds, where $0 \leq \sigma \leq 1$. *A 1-stable subpopulation is called a* stable *subpopulation.*

Remarks 5. The following remarks on subpopulations stability can be made:

(i) A σ-ustable subpopulation is a subpopulation for which (1) does not hold. In Roaming technique 1-unstable subpopulations evolve in isolation until they become stable;
(ii) The best individual in a stable subpopulation can be considered a potential local optima.

3.2 The Archive

Roaming technique uses an external population called the *archive* to store detected potential optima.

Consider a stable subpopulation P and x^* the best individual in P. It is reasonable to suppose that a potential optimum x^* can be a local optimum or can be very close to a local optimum.

A solution x^* is added to the archive only if it represents a new local optima, or it is better than another local optima that is already in the archive and placed on the same peak .

In order to ensure this, for each individual a in the archive, the global minimum min of the fitness function is calcultated on the domain delimited by x^* and a. If the minimum min indicates there is a 'valley' between x^* and every memeber of the archive, then add x^* to the archive. If not, then there exist an individual a in the archive such that x^* and a are located on the same peak. In this case x^* is compared with a and only the best of them will remain in the archive. If $x^* \succ a$ then x^* replaces a in the achive. If not, x^* is not added to the archive.

Remark 6. The minimum of the fitness function on the domain delimited by x^* and a can be calculated using a simple genetic algorithm or an evolution strategy. It is not even neccesary to calculate the minimum, the seach can be stoped when the first individual that is 'worst' than x^* and a is found, or after a given number of generations.

Remark 7. Most evolutionary approaches to the multimodal optimization problem make use of a distance measure. Because of that, ussually a parameter depending in some manner on the distance between the optima has to be used. Within Roaming technique the use of the distance, as well as the use of a distance-depending parameter has been eliminated .

3.3 Roaming Subpopulations

Consider a potential optimum x_i^* has been added to the archive. To avoid the search process to get stuck on a detected optima, the search performed by the subpopulation P_i has to be redirected towards other regions of the search space. In this respect, RS-stable subpopulations are selected to spread, where RS is a parameter of the algorithm.

Subpopulations selected for spreading are called *Roaming Subpopulations*. The roaming is realized using variation operators acting on subpopulations.

The next generation $P(t+1)$ will be composed of the roamed subpopulations and the offspring $P_i^{'}(t)$ of the subpopulations $P_i(t)$ that have not been selected to roam.

3.4 Roaming Algorithm

Roaming algorithm may be outlined as follows:

Roaming Algorithm

Input: N - subpopulations number
$Popsize$ - subpopulation size
$Ngen$ - maximum number of generations
δ - archive parameter
RS - roaming threshold
p_c, p_m -crossover probability and mutation rate

Output: $Archive$ - the set of local optima

Step 1. *Initialization:* $t := 0$; initialize $P_i(t)$ for each $i = 1, \dots N$ by randomly generating $popsize$ number of individuals; $Archive := \emptyset$.

Step 2. Evaluate each individual x in each subpopulation $P_i(t)$ by computing its fitness $eval(x)$;

Step 3. Evolve each subpopulation $P_i(t)$ one iteration. Let $P_i^{'}(t)$ be the resulting offspring subpopulation.

Step 4. Evaluate each individual x in $P_i^{'}(t)$.

Step 5. For each subpopulation $P_i(t)$ calculate:
a) The best individual x_i^*;
b) The stability measure $SM(P_i(t))$ using Definition 2.

Step 6. For each 1-stable subpopulation $P_i(t)$ try to add x_i^* to the $Archive$.

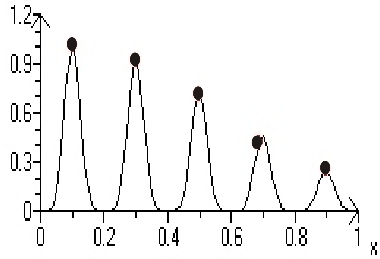

Fig. 1. Function f_1 - the archive after 10 generations

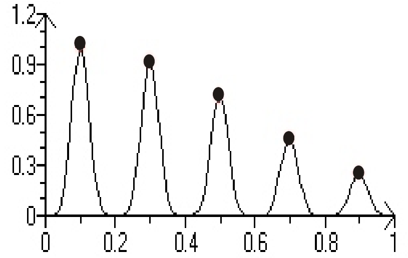

Fig. 2. Function f_1 - the archive after 75 generations

Step 7. For each $i = 1, \ldots N$ do if $SM(P_i(t)) \geq RS$ then consider $P_i(t)$ to be a roaming subpopulation;

Step 8. Migrate all roaming subpopulations using strong mutation with $rate = 1$

Step 9. Set $P(t+1) = \{P_i(t) \mid P_i(t)$ is a Roaming Subpopulation$\} \cup \{P'_i(t) \mid P_i(t)$ is not a Roaming Subpopulation$\}$; $t = t + 1$. If $t < Nrgen$ then go to step 2, else stop.

4 Experimental Results

Roaming Algorithm has been tested for several standard functions. In this section the following functions are considered:

$f_1(x) = e^{-2\ln(2)\left(\frac{x-0.1}{0.8}\right)^2} \sin^6(5\pi x)$, $x \in [0, 1]$,

$f_2(x) = \ln(x) \cdot (\sin(e^x) + \sin(3x))$, $x \in (0, 4]$,

Function f_1 is a standard test function for multimodal techniques. Funnction f_2 is presented to illustrate the fact that the algorithm works for functions with unevenly spaced optima.

The parameters used to run the algorithm for both functions f_1 and f_2 are: subpopulation number 15, subpopulation size 10, number of generations 75 and RS 0,8.

The results presented here are averaged over 10 runs. At each run, the roaming algorithm detected all optima of the functions f_1 and f_2.

Evolution of the archive content for function f_1 the is presented in Fig. 1 and 2. Roaming algorithm detects the peaks of the function at early stage of the search process. This can be noticed also for the function f_2 in Fig. 3.

Figure 4 presents the final achive for function f_2. The number of optima stored in the archive for each function in presented in Table 1.

5 Conclusions and Future Work

A new evolutionary technique for multimodal optimization called Roaming is proposed. Roaming uses a number of roaming subpopulations in order to detect

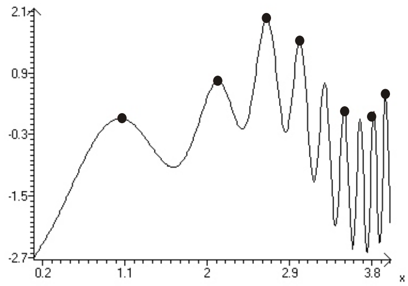

Fig. 3. Function f_2 - the archive after 10 generations

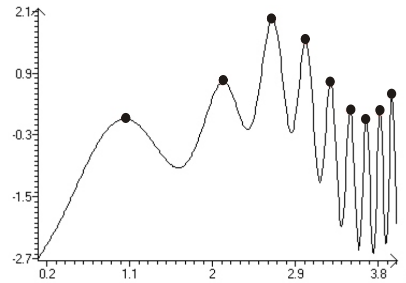

Fig. 4. Function f_2 - the archive after 75 generations

Table 1. Number of peaks detected for functions f_1 and f_2

Function	Number of generations	Number of detected peaks
f_1	75	5
f_2	75	9

multiple optima. A measure for the stability of a sub-population is introduced in order to asses whether a subpopulation has located an optimum or not.

Subpopulations evolve in isolation until an optimum is detected. Detected optima are saved into an archive and the corresponding subpopulations are spread towards new promissing regions of the search space.

Numerical examples are presented to illustrate the efficiency of the technique proposed.

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