A Strongly Interacting Dynamic Particle Swarm Optimization Method

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A new direct search method using only function values is proposed for finding a local minimizer of a real valued function $f(\mathbf{x})$, $\mathbf{x} = [x_1, x_2, ..., x_n]^T \in R^n$. This method is a synthesis of two unconventional trajectory methods for unconstrained minimization. The first is the *dynamic* method of Snyman [2], and the second method is the particle swarm optimization (PSO) method of Eberhardt and Kennedy [1].

In the new (DYN-PSO) method, the minimization of a function is achieved through the dynamic motion of a *strongly interacting* particle swarm, where each particle in the swarm is simultaneously attracted by all other particles located at positions of lower function value.

Computation of particle trajectories: The DYN-PSO method is started by generating at time $t = 0$, a swarm of np particles, each of unit mass, with random initial positions denoted by $\mathbf{x}\{i\}(0) = \mathbf{x}^0\{i\},\ i = 1, 2, ..., np$. Initially, at $t = 0$, these particles all have zero velocities, i.e. $\mathbf{v}\{i\}(0) = \mathbf{v}^0\{i\} = \mathbf{0}$. We now postulate that at time t each particle i experiences a force $\mathbf{a}\{i\}(t)$. The explicit analytical form of the force law giving $\mathbf{a}\{i\}(t)$ is given below. Thus the trajectories $\mathbf{x}\{i\}(t)$ of the particles are given by the solution to the system of *initial value problems*:

$$\ddot{\mathbf{x}}\{i\}(t) = \mathbf{a}\{i\}(t) \quad \text{for} \quad i = 1, 2, \ldots, np \tag{1}$$

with random initial positions $\mathbf{x}\{i\}(0) = \mathbf{x}^0\{i\}$ and zero initial velocities. In practice these equations are solved numerically by discretizing the time interval into time steps δ, and computing approximations $\mathbf{x}^k\{i\}$ to $\mathbf{x}\{i\}(t_k)$ at discrete time mesh points $t_k = k\delta,\ k = 0, 1, 2, \ldots$

The interacting force law: The explicit force law that we assume here dictates that the force of attraction experienced by a particle at higher function value due to a particle at a lower function value position, be equal to the difference between the respective function-values, divided by their distance of separation. A stochastic element is introduced by randomly perturbing the direction of action of the attracting force. More explicitly, at each iteration k the resultant force exerted on particle i at position $\mathbf{x}^k\{i\}$ with function value $f(\mathbf{x}^k\{i\})$, by the other $np - 1$ particles at positions $\mathbf{x}^k\{j\}$, $j = 1, 2, ..., np,\ j \neq i$, with corresponding function values $f^k(\mathbf{x}\{j\})$ are given by the force $\mathbf{a}^k\{i\}$ with components

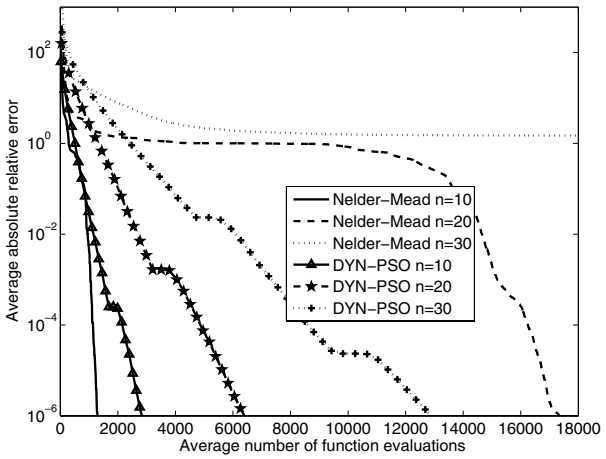
$$a_m^k\{i\} = \sum_{\substack{j=1 \\ j\neq i}}^{np} [x_m^k\{j\} - x_m^k\{i\}] c_j 2(rd)/\|\mathbf{x}^k\{j\} - \mathbf{x}^k\{i\}\| \tag{2}$$

for $m = 1, 2, \ldots, n$. Here c_j=max$[0, f(\mathbf{x}^k\{i\}) - f(\mathbf{x}^k\{j\})]$ and rd is an independent random number in [0,1].

In computing the trajectories of the particles $\mathbf{x}^k\{i\},\ i = 1, 2, ..., np$, for $k = 0, 1, 2, ...$, the function values $f_k(i) = f(\mathbf{x}^k\{i\})$ at $\mathbf{x}^k\{i\}$ are monitored at each iteration k so that the best (lowest) function value $f_b\{i\}$ and corresponding best position $\mathbf{x}^b\{i\}$ along each trajectory i are recorded. The overall globally best function value $f_g\{i\} = \min_i(f_b\{i\})$ and corresponding position $\mathbf{x}^g$ are also recorded.

An energy dissipation strategy is also applied to ensure local descent of a particle and the overall collapse of the swarm to a local minimum by extracting kinetic energy from a particle whenever it moves "up-hill".

The newly proposed DYN-PSO algorithm with $np = n+1$ was tested on a set of eight extended test problems with n ranging from 10 to 30. Typically its behaviour compared to that of the well-known Nelder-Mead algorithm, is as shown in the figure below for the homogeneous quadratic function $f(\boldsymbol{x}) = \sum_{i=1}^{n} ix_i^2$.



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