On Efficient Deployment of High-end Sensors in Large-scale Heterogeneous WSNs

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***Abstract*—Many complex sensor network applications require the use of a large number of low-end sensors to achieve quality through quantity. Deploying a relatively small number of high-end sensors in the region to gather and forward sensor data to the base station is generally considered as an efficient and scalable way to facilitate the management and operation of large-scale sensor networks. The number and location of high-end sensors do not only affect the network deployment cost but also the total energy consumption for data communication. We investigate the problem of deploying a minimum set of high-end sensors to collect the measurements of all low-end sensors with a minimum amount of energy consumption. We propose a heuristic algorithm, Distance- and Connectivity-based H-sensor Deployment to solve this problem. The simulation results illustrate the performance superiority of the proposed algorithm in comparison with two greedy schemes.**

Keywords: Heterogeneous sensor networks, sensor deployment, energy consumption

I. Introduction

Extensive studies have shown that homogeneous Wireless Sensor Networks (WSNs) impose a fundamental limit on application performance and system scalability. One important factor that causes such limitations is the Uneven Energy Consumption (UEC) problem among sensor nodes, which is inherent for sensor networks with the many-to-one traffic pattern and multi-hop communications.

To address this UEC problem, many researchers shifted their research efforts to a new type of WSN that consists of a large number of resource-limited low-end sensors, referred to as L-sensors, and a small number of resource-rich high-end sensors, referred to as H-sensors. In such large-scale Heterogeneous Sensor Networks (HSNs), L-sensors are often *a priori* deployed in a region of interest in a random manner (airborne, dropped by mobile vehicles, etc.) for environmental sensing and may have different energy levels[1]; while H-sensors are typically deployed at strategically determined locations for collecting and forwarding sensor data to the Base Station (BS). Obviously, the number and location of H-sensors as well as the routing scheme that determines which H-sensor to send data for each L-sensor, do not only affect the network deployment cost but also the total energy consumption for data communication, which affects the network lifetime to a large degree.

[1]The energy levels of L-sensors may vary in the very beginning or after a certain period of operation due to many factors including battery volume, geographical location, environmental interference, etc.

In this paper, we investigate the problem of deploying a minimum set of H-sensors to collect the environmental measurements of all existing L-sensors with a minimum amount of energy consumption. We formulate this problem as a multi-objective optimization problem and propose a heuristic algorithm, *Distance- and Connectivity-based H-sensor Deployment* (DCHD), based on rigorous geometric reasoning and deductive analysis. We also design two other heuristics based on greedy strategies, Distance-based Greedy (DG) and Coverage-based Greedy (CG), where DG provides the lower bound of the total energy cost (TEC). The performance superiority of the proposed algorithm is justified by the simulation results in comparison with the greedy schemes.

The rest of the paper is organized as follows. Related work is described in Section II. The problem is formulated in Section III. In Section IV, we propose a heuristic solution. Implementation details and performance evaluations are presented in Section V.

II. Related Work

The deployment of H-sensors has been studied in a variety contexts [1]–[3]. Most of these studies assume that the number of H-sensors is known *a priori*. The classical energy-efficient algorithm, *Low-Energy Adaptive Clustering Hierarchy* (LEACH) [1] assumes one cluster head for each cluster and employs randomized rotation of cluster heads to evenly distribute energy consumption among the sensors in the network.

Closely related to the H-sensor deployment problem is the relay node deployment problem. Some works along this line assume controllable node locations so that nodes can be deployed anywhere. Lloyd *et al.* achieved a prolonged network lifetime by deploying a

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small number of costly but more powerful relay nodes in two different communication models [4]. Tang *et al.* formulated two optimization problems of relay node deployment [5]: Connected Relay Node Single Cover (CRNSC) and 2-Connected Relay Node Double Cover (2CRNDC) problems. Our work differs from theirs in that we consider random sensor distributions while they consider uniform distributions. In other scenarios, sensor deployment locations are constrained in a set of candidate sites. Misra *et al.* studied the uncontrollable deployment of relay nodes in [6]. Dasgupta *et al.* presented the sensor deployment problem for maximum lifetime with coverage constraints and proposed an algorithm in [7].

The main differences between our work and aforementioned ones lie in several aspects. We consider two deployment objectives: (i) minimize the number of H-sensors to reduce deployment cost and (ii) minimize the TEC needed for collecting data from all L-sensors. We consider unrestricted deployment locations and employ different energy cost models for communications from L-sensors to H-sensors and from H-sensors to the BS.

III. Cost Models and Problem Formulation

We formulate a multi-objective optimization problem of minimizing the number and TEC of H-sensors to be deployed in a given region, where n_L L-sensors L_i with an identical radio radius r and different Initial Energy (IE) supply IE_{L_i}, $i = 0, 1, \ldots, n_L - 1$, are already randomly deployed. The location of each L-sensor is denoted as (x_{L_i}, y_{L_i}). A BS is deployed somewhere inside or outside the region. We investigate the problem of strategically deploying a set of H-sensors with an identical radio radius R and identical initial energy supply IE_H to collect data from all the L-sensors in the region, where $R \gg r$. The H-sensors are exempt from environmental sensing. We assume that (i) each L-sensor directly sends data to its corresponding H-sensor(s) with a constant amount of transmission energy cost denoted as EC_{LtoH}, and (ii) each H-sensor incurs a d^2 energy cost on one-hop communication, where d is the Euclidean distance between the H-sensor and the BS. The energy cost for sending one data packet from an H-sensor to the BS is determined by the following free space (fs) energy cost model [2]: $EC_{HtoBS} = E_{elec} + \varepsilon_{fs} * d_{i,BS}^2$, where E_{elec} denotes the energy for driving the electronics, which depends on various factors including digital coding, modulation, filtering and spreading of the signals; ε_{fs} is the coefficient for calculating the amplifier, which depends on the Euclidean distance $d_{i,BS}$ between an H-sensor H_i and the BS. We use RE_L and RE_H to represent the Residual Energy (RE) of L-sensors and H-sensors, respectively, during the network operation.

The deployment objective is to strategically place a minimum set of H-sensors to collect and forward the data of all L-sensors to the BS with minimum TEC. We further assume that the energy consumptions for environmental sensing at L-sensors and data receiving at H-sensors are negligible compared to the energy cost for data transmission. After H-sensors are deployed, each L-sensor sends a data packet to the BS through the assigned H-sensor(s) in each round. Note that an L-sensor may switch to different H-sensors at different times and the H-sensor routing information is stored at each L-sensor.

Since the total energy supply of each L-sensor and the energy cost for sending one packet from an L-sensor to an H-sensor are known, we can calculate the total number of data packets each L-sensor can generate. Similarly, the total number of data packets that each H-sensor should forward to the BS is also known once the location of the H-sensor is determined.

IV. Problem Analysis and Algorithm Design

A. Problem Analysis

H-sensors must be deployed at certain locations such that each L-sensor is able to connect to at least one H-sensor. In other words, H-sensors must be placed within the radio coverage disks of L-sensors. A simplified case of the H-sensor deployment problem without considering the energy consumption minimization objective where H-sensors and L-senors have the same radio radius is to find a minimum number of H-sensor radio coverage disks to cover all the L-sensors, which is known as the NP-complete *Minimum Geometric Disk Cover* (MGDC) problem [8]. The multi-objective H-sensor deployment problem is more challenging and we design heuristic algorithms to this problem.

B. Geometric Calculation of Closest Points

To minimize TEC, H-sensors must be deployed as close to the BS as possible; meanwhile any L-senor must be able to reach at least one H-sensor. We define several terms to facilitate the explanation of our algorithms.

Definition 1: **Division** is a minimum enclosed area intersected by the radio coverage disks of L-sensors and bounded by a set of basic arcs[2]. By "minimum", we mean that there does not exist any arc inside a division, or in other words, a division cannot be further divided by any arc into smaller divisions.

Definition 2: **Coverage Degree (CovDeg)** is the least number of L-sensors that can connect to an H-sensor, which is deployed inside a division or on its boundary.

Definition 3: **Connectivity Degree (ConDeg)** is the largest number of H-sensors to which an L-sensor can connect.

Definition 4: **Closest Point** is a point in a division that is the closest one to the BS compared to any other points inside or on the boundary of this division.

[2]A basic arc is an arc that is not intersected by any other arcs.

Definition 5: **Corner Point** is an intersection point of the boundary arcs of a division.

Each division is associated with a coverage degree and each L-sensor is associated with a connectivity degree. Note that we consider the "least number" for coverage degree in Definition 2 because the number of connected L-sensors may vary depending on whether the H-sensor is deployed inside the division or on its boundary.

Since the energy cost from an H-sensor to the BS is in proportion to the square of the Euclidean distance d between the H-sensor and the BS, the closest point of each division is a good candidate location for deploying the H-sensor. Proposition 1 is given below to find the closest point of each division. We assume that the BS is not located inside the coverage disk of any L-sensor, i.e. no L-sensor can directly connect to the BS.

Proposition 1: Given a division and the BS, the closest point of the division is the shortest one to the BS among the two types of points: (i) the intersection point between any boundary arc and the line that connects the BS and the center of the circle that contributes this boundary arc, and (ii) the corner points.

The correctness of Proposition 1 can be derived from a universal mathematical theorem that the closest point on a circle to a point P outside this circle is the intersection point of the circle and the line that connects p and the center of this circle. For example, as shown in Fig. 1, we consider the closest point of a division formed by four solid arcs contributed by Circles 1, 2, 3, and 4, whose centers are denoted as C1, C2, C3, and C4, respectively. In the left figure, there are two lines intersecting with the division: the first line connecting the BS and C3 intersects with the arcs provided by Circles 2 and 3, and the second line connecting the BS and C2 intersects with the arcs provided by Circles 1 and 3. According to Proposition 1, only the intersection point between the first line and Circle 3 needs to be considered. After a further comparison with all four corner points, we conclude that the closest point of this division should be the intersection point on the first line. In the right figure where the BS has been moved to a different location, there is no valid intersection point between the lines and the arcs, and the closest point of this division must fall on one of the corner points[3]. Obviously, the right-top corner point is the closest point to the BS in this case.

Identifying all divisions is a nontrivial and critical step since the proposed H-sensor deployment algorithms are based on the precalculation of divisions. A brief description of the algorithm we use to calculate and verify all valid divisions is provided in Alg. 1.

[3]If there are multiple corner points with the same closest distance to the BS, one of them is arbitrarily chosen to be the closest point of the division.

Algorithm 1 Division Calculation

Input: n_L L-sensors L_i with the same radio radius r and different locations (x_i, y_i), $i \in n_L$, are deployed in a WSN.
Output: All valid divisions.

Define *Cirs*, *Arcs* and *Divs* as the circle, arc and division list, respectively;
Calculate all the intersection points of circles using L-sensor as center and its radio r as radius;
Record the list of intersection points for each circle in *Cirs*;
for all circles $\in$ *Cirs* **do**
  if the number of intersection points is equal to 0 or 1 **then**
    Add this circle to *Divs*;
  else
    Add the arcs of this circle which cannot be further divided to *Arcs*;
    Record original endpoints of each arc as v_1 and v_2 in clockwise direction;
for all arcs $\in$ *Arcs* **do**
  Construct segment seg_1 such that the endpoints are the v_1 and the midpoint of the arc in clockwise direction;
  Construct segment seg_2 such that the endpoints are the midpoint of the arc and the v_2 in clockwise direction;
for all arcs $a_{start} \in$ *Arcs* **do**
  for all arcs who share the endpoint v_2 of a_{start} **do**
    Set arc a_{tmp} as the arc who has the minimum angle in clockwise direction between the segments that share the endpoint v_2 of the arc a_{start};
    Record endpoints of the arc a_{tmp} as v'_1 and v'_2 in the tracing order;
  while arc $a_{tmp} \neq a_{start}$ **do**
    for all arcs who share the endpoint v'_2 of a_{tmp} **do**
      Set arc a_{end} as the arc who has the minimum angle in clockwise direction between the segments that share the endpoint v'_2 of the arc a_{tmp};
      Record endpoints of the arc a_{end} as v'_1 and v'_2 in the tracing order;
    $a_{tmp} = a_{end}$;
  if this division $\notin$ *Divs* **then**
    Add this division and all its component arcs to *Divs*, including arc a_{start};
  for all arcs who share the endpoint v_2 of a_{start} **do**
    Set arc a_{tmp} as the arc who has the minimum angle in counterclockwise direction between the segments that share the endpoint v_2 of the arc a_{start};
    Record endpoints of the arc a_{tmp} as v'_1 and v'_2 in the tracing order;
  while arc $a_{tmp} \neq a_{start}$ **do**
    for all arcs who share the endpoint v'_2 of a_{tmp} **do**
      Set arc a_{end} as the arc who has the minimum angle in counterclockwise direction between the segments that share the endpoint v'_2 of the arc a_{tmp};
      Record endpoints of the arc a_{end} as v'_1 and v'_2 in the tracing order;
    $a_{tmp} = a_{end}$;
  if this division $\notin$ *Divs* **then**
    Add this division and all its component arcs to *Divs*, including arc a_{start};
for all divisions $\in$ *Divs* **do**
  if all its component arcs are concave **then**
    if not all the intersection points are inside or on the same circle **then**
      Remove the division and its arcs from *Divs*;
  if all its component arcs are convex **then**
    for all arcs **do**
      if other arcs on the same circle as any one of those convex arcs intersect with the division **then**
        Remove the division and its arcs from *Divs*;
return all valid divisions *Divs*;

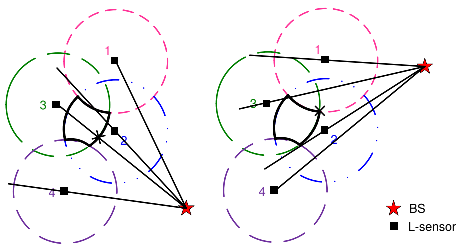


Fig. 1. Two types of closest points.

We identify all the divisions with coverage degree ≥ 1 based on geometric analysis using Alg. 1 and then calculate the closest point of each division to the BS based on Proposition 1. The obtained closest points are a complete set of candidate deployment locations, a subset of which might be chosen for deploying H-sensors. We also calculate the coverage degree of each division and the connectivity degree of each L-sensor.

C. Algorithm Design for H-sensor Deployment

In the H-sensor deployment problem, we wish to minimize the number of deployed H-sensors to save deployment cost and minimize the distance between each H-sensor and the BS to reduce communication energy cost. Based on the closest point calculation for each division using Proposition 1, we propose a heuristic algorithm, *Distance- and Connectivity-based H-sensor Deployment* (DCHD), which takes into consideration both the distance between H-sensors and the BS and the connectivity of L-sensors. Once H-sensors are deployed, we consider a simple routing (or L-H sensor assignment) scheme where an L-sensor sends data packets to an H-sensor that is the closest to the BS among all the H-sensors that can be reached by this L-sensor.

For performance comparison purposes, we also present two naive greedy algorithms, Distance-based Greedy (DG) and Coverage-based Greedy (CG). DG provides the lower bound on the TEC of deployed H-sensors for communications but may require deploying more H-sensors than necessary, hence causing a large amount of unused or wasted energy.

1) DCHD Algorithm: DCHD algorithm identifies a subset of locations to deploy H-sensors from all candidate locations to minimize the number and the TEC of the deployed H-sensors. L-sensors are assigned to H-sensors in an increasing order according to their connectivity degrees[4]; in other words, L-sensors with the lowest connectivity degrees are scheduled first since they have the most stringent constraint on choosing the best H-sensors. Furthermore, H-sensors that are already deployed with residual energy $RE > 0$ are always considered first to make full use of their resources.

The pseudocode of DCHD is shown in Alg. 2, where lines 1-6 define some data structures, lines 7-8 perform initialization, and the rest describe the DCHD algorithm. On lines 10-11, the L-sensor with the smallest connectivity degree is selected. On lines 12-19, the L-sensor sends packets to the H-sensors which have been deployed in the previous steps but still have residual energy. On lines 20-26, more H-sensors are deployed to receive packets if the L-sensor still has energy left but the previously deployed H-sensors have run out of energy.

Algorithm 2 DCHD

Input: A WSN where n_L L-sensors L_i with different IE_{L_i}, $i \in [0,\ n_L - 1]$, are deployed, n_{div} candidate locations (divisions) for deploying H-sensors with identical IE_H, and the BS.
Output: The minimum number n_H of H-sensors H_j, their locations (x_{H_j}, y_{H_j}), $j \in [0,\ n_H - 1]$, and minimum TEC.

1: Define UAL as a set of unassigned L-sensors;
2: Define HRE as a set of deployed H-sensors with $RE_H > 0$;
3: Define $d(H_j)$ as the distance between the candidate location (x_{H_j}, y_{H_j}), $j \in [0,\ n_{div} - 1]$, and the BS;
4: Define $ConHS(L_i)$ as a set of candidate H-sensors with $RE_H > 0$, to which L-sensor L_i is connected;
5: Define $LHTab$ as a 2D table that stores all the L-sensors and their corresponding H-sensors for data forwarding;
6: Define $DHTab$ as a 2D table that stores the actually deployed H-sensors and their locations;
7: Initialize $RE_{L_i} = IE_{L_i}$, for all $i \in [0,\ n_L - 1]$;
8: UAL = all L-sensors, $CountH = 0$, $HRE = \emptyset$;
9: **while** $UAL \neq \emptyset$ **do**
10:     **if** $ConDeg(L_k)$ = Min($ConDeg(UAL)$), $L_k \in UAL$ **then**
11:         Set L_k as assigned, remove L_k from UAL;
12:     **while** $RE_{L_k} > 0$ **do**
13:         **for all** $H_t \in ConHS(L_k)$ and $H_t \in HRE$ **do**
14:             Find H_t with the shortest Euclidean distance to the BS;
15:         Assign L_k to H_t, update L_k, H_t and $LHTab$;
16:         **if** $RE_{H_t} = 0$ **then**
17:             Remove H_t from $ConHS(L_k)$ and HRE;
18:         **if** $H_t \notin HRE$ for all $H_t \in ConHS(L_k)$ **then**
19:             Break;
20:     **if** $RE_{L_k} > 0$ **then**
21:         Find $H_t \in ConHS(L_k)$ such that $d(H_t) = \text{Min}(d(ConHS(L_k)))$;
22:         Deploy n H-sensors at the location of H_t to forward the rest data packets from L_k, where $n =$ packets left / packets that can be forwarded by one H-sensor;
23:         $CountH = CountH + n$;
24:         Update $LHTab$ and $DHTab$;
25:         **if** $RE_{H_{n-1}} > 0$ **then**
26:             Add H_{n-1} to HRE;
27: **return** $CountH$, $LHTab$ and $DHTab$;

2) DG and CG Algorithms: DG algorithm always chooses the division, whose closest point is the closest one to the BS among all the closest points and does not have any H-sensors deployed, to deploy a minimum sufficient number of H-sensors to forward all the packets from the covered L-sensors to the BS. CG algorithm chooses a division to deploy H-sensors that can receive the largest number of data packets from L-sensors, i.e. cover as many L-sensors as possible.

[4] When an L-sensor is assigned to an H-sensor, the L-sensor sends data to the H-sensor for forwarding to the BS.

V. Implementation and Experimental Results

The proposed DCHD algorithm and the other two greedy algorithms, DG and CG, are implemented in C++ and tested on a Windows XP desktop equipped with a 3.0 GHz CPU and 2 Gbytes memory. We conduct an extensive set of experiments on a large number of simulated sensor networks that are created by varying: (i) the number of initially deployed L-sensors and (ii) the distribution and radius of L-sensors. The sensor network region is fixed in a square planar area of $200 \times 200\ m^2$, whose left bottom corner is set as the origin $(0,0)$. The BS is located somewhere outside the sensor network region. We assume that each H-sensor has 5 J initial energy and one fixed-sized data packet transferred from an L-sensor to its corresponding H-sensor costs 500 μJ.

A. Comparison for Different Radiuses of L-sensors

We compare the minimum number and TEC of deployed H-sensors calculated by three algorithms, DCHD, DG and CG, in the experiments performed on two randomly generated simulated sensor networks: a small network with only 10 L-sensors and a large one with 100 L-sensors. We measure the performance of these three algorithms in terms of the number and TEC of deployed H-sensors as we increase the radius of the deployed L-sensors from 0.5 to 3, 6, 10, 30, 50, 70, 90, 100 to 120 m, indexed from 1 to 10, respectively. The performance curves measured from the large network are plotted in Fig. 2. The performance curves from the small network are qualitatively very similar and are not plotted due to space limit. In Fig. 2, we observe that although DG achieves less TEC than DCHD, it requires deploying much more H-sensors, resulting in a large amount of unused or wasted energy; similarly, CG might also deploy more H-sensors than DCHD since it consumes much more energy due to the long distance from the H-sensors to the BS.

B. Comparison for Different Numbers of L-sensors

We further study the performance of these three algorithms as we vary the network size from small to large scales as shown in Fig. 3. We fix the L-sensor's radius to be 30 m and randomly generate 10 problem cases of test networks with different numbers of deployed L-sensors ranging from 10, 50, 100, 150, 200, 250, 300, 350, 400, to 500, indexed from 1 to 10, respectively. We project the 3D performance comparison in Fig. 3(a) to a 2D comparison on TEC in Fig. 3(b) and the number of deployed H-sensors in Fig. 3(c). We observe that DCHD achieves a very close performance to DG in terms of TEC but requires much less H-sensors than DG and CG.

To maximize the utilization of resources (the energy of H-sensors), a good deployment scheme would place an appropriate number of H-sensors with a sufficient but

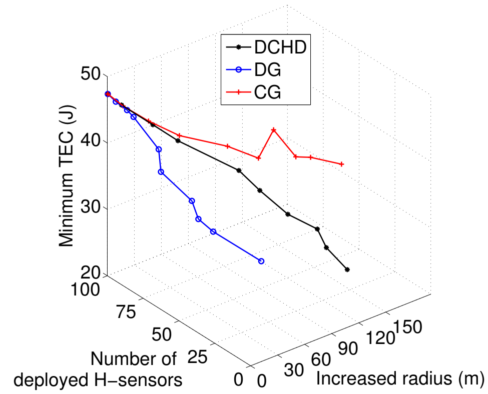


Fig. 2. Performance comparison of 3 algorithms as L-sensor's radius increases in a large network with n = 100 L-sensors.

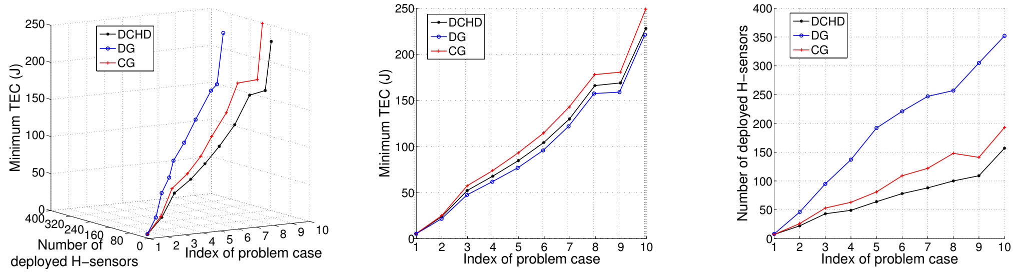
not superfluous amount of energy to forward data packets from all L-sensors since any leftover or unused energy beyond the network operation lifetime is considered as a waste. In Fig. 4, each bar represents the total energy of deployed H-sensors, the lower part of which is the total energy used to forward all the packets in the network and the upper part of which is the total energy left. We observe that DCHD (the first bar in each problem case) requires the minimum total deployed energy[5] and results in the minimum residual energy among three algorithms. DG deploys much more H-sensors, about two thirds of whose energy is unused or wasted.

In order to investigate the robustness of our algorithm, we further compare the mean value and standard deviation of TEC calculated by three algorithms. Similar to the previous case, we fix the L-sensor's radius to be 50 m and define 10 problem sizes with 10, 30, 50, 80, 100, 120, 150, 200, 250, and 300 L-sensors, indexed from 1 to 10, respectively. We randomly generate 10 instances of different network topologies for each problem size and plot the performance measurements and statistics in Fig. 5. We observe that DCHD achieves the smallest standard deviations among three algorithms and its mean values are less than those of CG.

C. Visualization of a Small Example

For illustration purposes, we lay out the H-sensors determined by three algorithms in a small network with 20 L-sensors of 35 m radius in Fig. 6, where the pentagram denotes the BS, the solid squares represent L-sensors, and the triangles, dots and circles indicate the locations of H-sensors deployed by DCHD, DG and CG, respectively. We observe that DCHD deploys the

[5] Since H-sensors have the same initial energy, the minimum total deployed energy results in a minimum number of H-sensors.



(a) (b) (c)

Fig. 3. Minimum number and TEC of H-sensors in 10 problem cases from small to large scales. (a) 3D comparison, (b) minimum TEC, (c) minimum number of H-sensors.

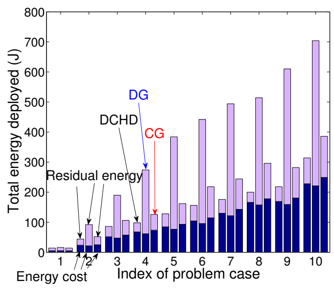


Fig. 4. Comparison of minimum energy cost and total energy allocated.

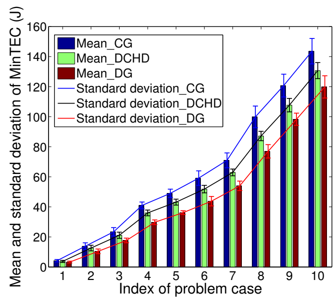


Fig. 5. Comparison of mean values and standard deviations of minimum TEC.

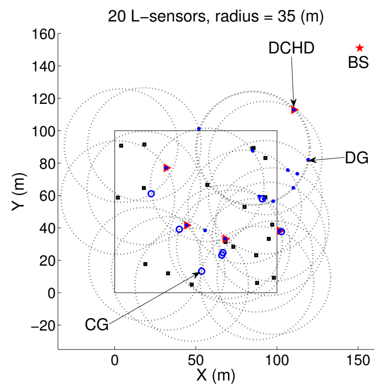


Fig. 6. Visualization of H-sensor layouts produced by 3 algorithms in a $100 \times 100\ m^2$ region.

least number of H-sensors (represented by triangles) at relatively more reasonable locations. Note that the H-sensor locations determined by DG contain all the H-sensor locations of DCHD, hence causing redundancy in deployment, which justifies our observation that DG has much more leftover or wasted energy as shown in the previous experiments.

VI. Conclusion

We tackled the problem of deploying a minimum set of H-sensors at appropriate locations to collect data packets from all L-sensors in the entire sensor network using minimum TEC. The simulation results illustrated the performance superiority of the proposed solution in comparison with the other two greedy algorithms. In the present work, we only considered one-hop communications from L-sensors to H-sensors and from H-sensors to the BS. We will consider multi-hop routing methods and the energy balance of H-sensors in our future work.

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