Delay and Energy Efficiency Tradeoffs for Data Collections and Aggregation in Large Scale Wireless Sensor Networks

Chao Ren†, Xufei Mao‡, Ping Xu‡, GuoJun Dai*, ZhanHuai Li†
†School of CS, Northwestern Polytechnical University, China. Email: renchao@cdgdc.edu.cn
‡Dept. of Computer Science, Illinois Institute of Technology, Chicago. Email: xmao3, pxu3@iit.edu
*Institute of Computer Application Technology, Hangzhou Dianzi Univ. China. Email: daigj@hdu.edu.cn

***Abstract*— In this paper, we study efficient data collection and aggregation problem in wireless sensor networks. We first propose efficient distributed algorithms for data collection problem with approximately the minimum delay, or the minimum number of messages to be sent by all wireless nodes, or the minimum total energy consumption by all wireless nodes respectively. For example, given an algorithm $\mathcal{A}$ for data collection, let ϱ_T, ϱ_M, and ϱ_E be the approximation ratio of $\mathcal{A}$ in terms of time complexity, message complexity, and energy complexity respectively. We then show that, for data collection, there are networks of n nodes and maximum degree Δ, such that $\varrho_M \varrho_E = \Omega(\Delta)$ for *any* algorithm. In addition, we analytically proved that all our proposed methods are either optimum or within constants factor of the optimum. We further present the message, energy, time complexity and studied the complexity tradeoffs for data aggregation problem.**

***Index Terms*— Time complexity, message complexity, energy, sensor networks, data collection, distributed algorithms.**

I. Introduction

In this paper, we study some fundamental complexity problems for data collection in wireless sensor networks. Given a set U of wireless sensor nodes distributed in a two dimensional or three dimensional space, we assume that a subset $V \subseteq U$ of nodes produce some data, *e.g.*, sampling temperature, moisture and so on. *Data collection* is to collect the set of data items A_i stored in each individual node $v_i \in V$ to the sink node (assuming v_0). We first design efficient algorithms whose complexity is asymptotically same as (or within a certain factor of) the complexity of the optimum for data collection. We further study various complexity tradeoffs and show that our method achieves the asymptotically optimum complexity tradeoffs. The complexity of a problem is defined as the worst case cost (time, message or energy) by the best algorithm.

To the best of our knowledge, we are the first to study the tradeoffs among the message complexity, time complexity, and energy complexity for data collection; we are the first to present lower-bounds (and matching upper-bounds for some cases) on the message complexity, time complexity, and energy complexity for data collection in wireless networks.

The main contributions of this paper are as follows. We design algorithms whose time complexity and message complexity are within constant factors of the optimum. The minimum energy data collection can be done using minimum cost shortest path tree. We further show that no data collection algorithm can achieve approximation ratio ϱ_M for message complexity and ϱ_E for energy complexity with $\varrho_M \cdot \varrho_E = o(\Delta)$. We then prove that our data collection algorithm has energy cost within a factor $O(\Delta)$ of the optimum while its time and message complexity are within $O(1)$ of the corresponding optimum. Thus, our method achieves the best tradeoffs among the time complexity, message complexity and energy complexity.

The rest of the paper is organized as follows. In Section II, we first present our wireless sensor network model, define the problems to be studied in this paper, and then briefly review the connected dominating set. We present several efficient methods for data collection in Section IV and we study the complexity tradeoffs of distributed data collection in Section V. Data aggregation is studied in Section VI. We review the related works in Section VII and conclude the paper in Section VIII.

II. Preliminaries

A. Network Model

We assume that there are $M+1$ wireless sensor nodes $U = \{u_0, u_1, u_2, \cdots, u_M\}$ that are deployed in a certain geographic region, where u_0 is the sink node. Each wireless sensor node corresponds to a vertex in a graph G and two vertices are connected iff their corresponding sensor nodes can communicate directly. We assume that links are "reliable", *i.e.*, the communication cost between two neighbor nodes is only 1. although in practice node v_i may need re-transmit several times. In some of the results, we further assume that all sensor nodes have a communication range r and a fixed interference range $R = \Theta(r)$. For simplicity, we may assume that $r = 1$ such that the communication graph G is a *Unit Disk Graph*(UDG).

Let $h(v_i, v_j)$ be the hop number of the minimum hop path connecting v_i and v_j in graph G, and $D(G)$ be the diameter of the graph. Here, we assume that $D(G) \geq 2$. If $D(G) = 1$, then the graph G is simply a completed graph and all questions studied in this paper can either be trivial or have been solved [8–10]. For a graph G, we denote its maximum degree as $\Delta(G)$. When each node v_i has n_i data items, we define the weighted degree, denoted as $\tilde{d}_{v_i}(G)$, of a node v_i in graph G as $n_i + \sum_{v_j : v_i v_j \in G} n_j$. The maximum weighted degree of a graph G, denoted as $\tilde{\Delta}(G)$, is defined as $\max_i \tilde{d}_{v_i}(G)$.

Each wireless node is able to monitor the environment, and collect some data (such as temperature). Assume that there is a subset of n wireless node $V \in U$ that collected some data. $A =$

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$\{a_1, a_2, \cdots, a_N\}$ is a totally ordered multi-set of N elements collected by all n nodes V. Here, N is the cardinality of set A. Each node v_i has n_i amount of raw data, denoted as $A_i \subset A$. Since A is a multi-set, we assume $A_i \cap A_j = \emptyset$ for $i \neq j$ and $A = \bigcup_{i=1}^n A_i$ Then $\langle A_1, A_2, \cdots, A_n \rangle$ is called a distribution of A at sites of V. We assume that one packet (*i.e.*, message) can contain one data item a_i, the node ID, plus additional constant number of bits, *i.e.*, the packet size is at the order of $\Theta(\log n + \log U)$, where U is the upper-bound on values of a_i. Such a restriction on the message size is realistic and needed, otherwise a single convergecast would suffice to accumulate all data items to the sink which will subsequently solve the problems easily. We consider a TDMA MAC schedule and assume that one time-slot duration allows transmission of exactly one packet.

If energy consumption is to be optimized, we assume that the *minimum* energy consumption by a node u to send data correctly to a node v, denoted as $E(u, v)$, is $c_1 \cdot \|u - v\|^\alpha$, where c_1 (normalized to 1 hereafter) and $\alpha \geq 2$ are constants depending on the environment. We assume that each wireless sensor node can dynamically adjust its transmission power to the minimum needed. Additionally, for a node v_i to send a data packet to a neighboring node v_j (with $v_i v_j \in G$), node v_j will also spend energy for receiving. In this paper, we assume that the receiving energy cost for a single packet is a fixed value E_r. We first assume that $E_r = 0$.

For data queries in WSNs, we often need build a spanning tree T of G first for pushing down queries and propagating back the intermediate results. Given a tree T, let $H(T)$ denote the height of the tree, *i.e.*, the number of links of the longest path from root to all leave nodes. The depth of a node v_i in T, denoted as $d_T(v_i)$, is the length of the path from the root to v_i. The subtree of T rooted at a node v_i is denoted as $T(v_i)$, the parent node of v_i is denoted as $p_T(v_i)$, and the set of children nodes of a node v_i is denoted as $\mathrm{Child}(v_i)$.

B. Problems and Complexities

We will mainly study the time complexity, message complexity, and energy complexity of data collection in WSNs.

The complexity measures we use to evaluate the performance of a given protocol are worst-case measures. The *message complexity* (and the *energy complexity*, respectively), of a protocol is defined as the maximum number of total messages (the total energy used, respectively) by all nodes, over all inputs, *i.e.*, over all possible wireless networks $\mathcal{G}$ of n nodes (and possibly with additional requirement of having diameter D and/or maximum nodal degree Δ) and all possible data distributions of A over V. The *time complexity* is defined as the lapsed time from the time when the first message was sent until the last message was received. The *lower bound* on a complexity measure (*e.g.*, message complexity) is the minimum complexity (*e.g.*, message complexity) required by *all* protocols that answer the queries correctly. The approximation ratio ϱ_T (resp. ϱ_M and ϱ_E) for an algorithm denotes the worse ratio of the time complexity (resp. message complexity and energy consumption) used by this algorithm compared to an optimal solution over all possible problem instances.

Here we assume that TDMA MAC is used for channel usage. Obviously, the complexity depends on the TDMA schedule policy $\mathcal{S}$. Let $X(v_i, t)$ denote whether node v_i will transmit at time slot t or not. Then a TDMA schedule policy $\mathcal{S}$ is to assign 0 or 1 to each variable $X(v_i, t)$. A TDMA schedule should be *interference free*: no receiving node is within the interference range of the other transmitting node. In other words, if the schedule is define for tree T, for any time slot t, if $X(v_i, t) = 1$, then $X(v_j, t) \neq 1$ for any node v_j such that $p_T(v_i)$ is within the interference range of v_j.

Data collection is an operation to collect the set of *raw* data items A from all sensor nodes to the sink node. It can be done by building a spanning tree T rooted at the sink v_0, and sending the data at every node v_i to the root node along the unique path in the tree. Clearly, the **message complexity** of data collection along T is $\sum_{i=1}^n n_i \cdot d_T(v_i)$. The **energy complexity**, defined as the total energy needed by all nodes for completing an operation, of data collection using T is $\sum_{i=1}^n [E(v_i, p_T(v_i)) \cdot \sum_{v_j \in T(v_i)} n_j]$.

The TDMA schedule should also be *valid* in the sense that every datum in the network will be relayed to the root. In other words, in tree T, when node v_i sends a datum to its parent $p_T(v_i)$ at a time slot t, node $p_T(v_i)$ should relay this datum at some time-slot $t' > t$. The largest time $\mathcal{D}$ such that there exists a node v_i with $X(v_i, \mathcal{D}) = 1$ is called the **time complexity** of this valid schedule. Time $\mathcal{D}$ is also called the *mark-span* of the schedule $\mathcal{S}$. Generally, a schedule $\mathcal{S}$ can be defined as assigning 0 or 1 to variable $X(v_i, t, k)$: it is 1 if and only if node v_i will relay datum a_k at time slot t. Clearly, a schedule $\mathcal{S}$ is *valid* for data collection of A using tree T, iff for every node v_i and time slot t, $\sum_{v_j \in \mathrm{Child}(v_i)} \sum_{b=1}^{t-1} X(v_j, b) + n_i \geq \sum_{b=1}^{t} X(v_i, b)$. Here $\sum_{v_j \in \mathrm{Child}(v_i)} \sum_{b=1}^{t-1} X(v_j, b) + n_i$ is the total number of data items node v_i has seen so far till time slot t and $\sum_{b=1}^{t} X(v_i, b)$ is the total number of data items that have been relayed by node v_i so far till time slot t. Then the time-complexity optimizing data collection problem is to find a spanning tree T and a *valid, interference-free* schedule $\mathcal{S}$ such that the mark-span is minimized.

III. Connected Dominating Set

A number of our methods will be based on a "good" connected dominating set (CDS) that has bounded degree $\mathbf{d}$ and bounded hop spanning ratio. Some definitions and properties of CDS have been studied well in [3]. We list some of them used in this paper and omit the proof.

Given a graph $G = (V, E)$, let $\mathbf{C} = (V_\mathbf{C}, E_\mathbf{C})$ be a connected dominating set of G where $V_\mathbf{C}$ is the set of dominators and connectors and $E_\mathbf{C}$ is the edges between dominators and connectors. For a node $v \in V_\mathbf{C}$, let $T_\mathbf{C}$ be a BFS tree of $\mathbf{C}$. For a node $v \in V \setminus V_\mathbf{C}$, we define a unique dominator $d(v)$ which is the one having the shortest hop distance to the sink v_0.

Definition 1—Data Communication Tree (DCT): For a graph G and its CDS, we define the data communication tree T as $T = (V, T_\mathbf{C} \cup \{\overline{vd(v)} \mid v \in V \setminus V_\mathbf{C}\}$.

Theorem 1: Let G and $\mathbf{C}$ be a graph and its CDS respectively. The DCT T has following properties:

1) $\Delta(T_\mathbf{C}) \leq \mathbf{d}$.
2) For any edge $e \in E_T$, let $I(e)$ be the set of edges in $T_\mathbf{C}$ that have interferences with e, then $|I(e)| \leq c \cdot \mathbf{d} \cdot \Delta(G)$ for some constant c depending on R/r.

Lemma 2: Given a *good* CDS of the graph G, data clustering can be done in time $O(\tilde{\Delta}(G))$.

IV. Efficient Data Collection

In this section, we design efficient methods for collecting data in wireless sensor networks.

A. Minimize Message

We first study the data collection with the minimum number of messages. When all links are reliable, clearly, we should collect any source data from a source node v_i to the sink node v_0 over the path with the minimum number of relay nodes, *i.e.*, with minimum hop number. Thus, the following theorem is straight forward.

Theorem 3: Data collection can be done with minimum number of messages $\sum_{i=1}^{n} n_i \cdot h(v_i, v_0)$ using a BFS tree with root v_0 if all links are reliable.

When links are not reliable, let $p(u, v)$ be the reliability of link (u, v), *i.e.*, with probability $p(u, v)$ the packet will be successfully transmitted over link (u, v). Assume that the link layer reliability is used. Then clearly, the number of expected transmissions over a link (u, v) is $1/p(u, v)$ for a packet. Thus, to minimize the total relays for sending a packet from its source v_i to the sink node v_0, the path Π must minimize $\sum_{(u,v)\in\Pi} \frac{1}{p(u,v)}$, which is just the least weighted path from v_i to v_0 when the weight of every link (u, v) is defined as $1/p(u, v)$. Thus, to minimize the messages for data collection, each data packet should be sent using the least weighted path (where the weight of a link (u, v) is $1/p(u, v)$) to the sink node. Let $\mathbf{p}(v_i, v_0)$ be the minimum weight of such shortest path. Let MRPT (maximum reliability path tree) be the tree rooted at v_0, formed of all these least weighted paths. Thus,

Theorem 4: Data collection can be done with minimum number of messages $\sum_{i=1}^{n} n_i \cdot \mathbf{p}(v_i, v_0)$ using a MRPT tree with root v_0 if a link (u, v) has a reliability $p(u, v)$.

B. Minimize Energy

We then study the data collection with the minimum energy cost. Apparently, for any element, it should follow the minimum energy cost path from its origin to the sink node v_0 in order to minimize the energy consumption. So minimizing the energy is equivalent to the problem of finding the shortest paths from the sink to all nodes (where the link cost is the its energy cost now), which clearly can be done in time $O(m + n \log n)$ for a communication graph of n nodes and m links. We call the tree formed by minimum energy path from the root to all nodes as the *minimum energy path tree (MEPT)*. Let $\mathbf{E}(v_i, v_0)$ be the energy cost of the path from v_i to v_0 with the minimum energy cost. Thus, we have

Theorem 5: Data collection can be done with minimum energy cost $\sum_{i=1}^{n} n_i \cdot \mathbf{E}(v_i, v_0)$ using a MEPT tree with root v_0 if a link (u, v) has an energy cost $E(u, v)$.

Clearly, when links are not reliable, we have to take into account the energy cost in retransmissions. In other words, we need use $E(u, v)/p(u, v)$ as the expected energy cost of a link (u, v). Obviously, the least cost path based routing is also optimum for minimizing the total energy cost of data collection if the receiving energy cost $E_r > 0$.

C. Minimize Time Delay

Then we study the time complexity of data collection. Notice that, the transmissions of nearby nodes should be in different time slots to avoid the interferences. We assume that all links are reliable hereafter.

Algorithm 1 presents our efficient data collection method based on a good CDS **C**. The constructed CDS has a degree at most a constant **d**, and similar to Theorem 1, all nodes in CDS can be scheduled to transmit once in constant $\beta = \Theta(\mathbf{d})$ time-slots without causing interferences to other nodes in CDS. We take β time-slots as one *round*.

First, the data elements from each dominatee node (a node not in **C**) are collected to the corresponding dominator node in the CDS **C**. Here the dominatee nodes that are one-hop away from the sink node v_0 will directly send the data to v_0. Notice that this can be done in time-slots $O(\tilde{\Delta}(G))$ by Lemma 2.

Now we only consider the dominator nodes and the breadth-first-search spanning tree $T_{\mathbf{C}}$ of nodes in CDS rooted at the sink v_0. Every edge in the tree $T_{\mathbf{C}}$ will be scheduled exactly once in each round. For simplicity, we do not schedule sending an element more than once in the same round. At every round, nodes in CDS push one data item to its parent node until all data are received by v_0.

Algorithm 1 Efficient Data Collection Using CDS

Input: A CDS **C** with bounded degree **d**, tree $T_{\mathbf{C}}$.

Every node v_i sends its data to its dominator node $d(v_i)$.
for $t = 1$ to N **do**
    for each node $v_i \in V_{\mathbf{C}}$ **do**
        If node v_i has data not forwarded to its parent node in $T_{\mathbf{C}}$, node v_i sends a new data to its parent in round t.

Theorem 6: Given a connected wireless network G, data collection can be done in time $\Theta(N)$ with $\Theta(\sum_{i=1}^{n} n_i h(v_i, v_0))$ messages.

Proof: For proof, please refer to [3]. ■

V. Complexity Tradeoffs for Data Collection

One may want to design a universal data collection method whose time-complexity, message-complexity and energy-complexity are all within constant factors of the optimum. Observe that Algorithm 1 is a constant approximation for both time-complexity and message-complexity. However, it is not a constant approximation for energy-complexity. Consider the following line network example: $n+1$ nodes are uniformly distributed in a line segment $[0, r]$; Sink v_0 is the leftmost node and node v_i is at position $i \cdot r/n$ and has one data item. Here we assume $r = 1$. See Figure 1 for illustration. Assume the energy cost for a link uv is $\|uv\|^2$. Then the minimum cost data collection is to let node v_i send all its data to node v_{i-1}. The total energy cost is $\sum_{i=1}^{n} i \cdot \frac{1}{n^2} \simeq 1/2$. While the energy cost of collecting data via CDS is $\sum_{i=1}^{n} (\frac{i}{n})^2 \simeq n/6$. On the other hand, the total number of messages of the minimum-energy data collection scheme is $n(n-1)/2$ and the time slots used by this scheme is also $\Theta(n^2)$; both of which are $\Theta(n)$ times of the corresponding minimum.

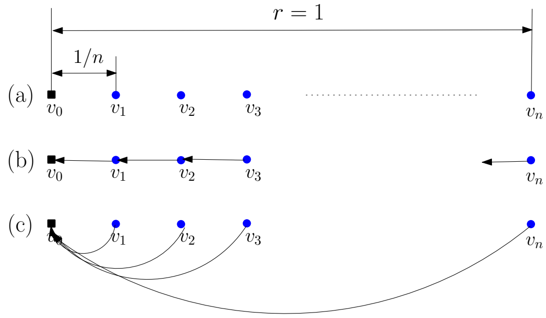

Fig. 1. Example: (a) a line network with $n+1$ nodes; (b) the minimum energy data collection tree; (c) the data collection tree via CDS, where v_0 is the only dominator.

Consider any data collecting algorithm $\mathcal{A}$. Let ϱ_M and ϱ_E be the approximation ratio for the message-complexity and energy-complexity of algorithm $\mathcal{A}$. We show that there are graphs of n nodes such that $\varrho_M \cdot \varrho_E = \Omega(n)$.

Lemma 7: Assume the energy cost for supporting a link uv is $\|uv\|^2$. For any data collection algorithm $\mathcal{A}$, there are graphs of n nodes, such that $\varrho_M \cdot \varrho_E = \Omega(n)$.

Proof: For proof, please refer to [3]. ■

Notice that we generally assumed that the energy cost for supporting a link uv is $\|uv\|^\alpha$. Then we can show that $(\varrho_M)^{\alpha-1}\varrho_E \geq \frac{n^{\alpha-1}}{2^{\alpha-1}}$. Notice that since $\varrho_E \geq 1$ and $\alpha \geq 2$, we have $(\varrho_M)^{\alpha-1}(\varrho_E)^{\alpha-1} \geq (\varrho_M)^{\alpha-1}\varrho_E \geq \frac{n^{\alpha-1}}{2^{\alpha-1}}$. Consequently, $\varrho_M \cdot \varrho_E \geq n/2$ still holds.

When we also take the maximum degree Δ into account, the above lemma implies the following corollary (the proof is essentially same).

Corollary 8: For any data collection algorithm $\mathcal{A}$, there are graphs of n nodes with maximum degree Δ, such that $\varrho_M \cdot \varrho_E = \Omega(\Delta)$.

The above lemma also implies that for any data collection algorithm $\mathcal{A}$, $\varrho_M \cdot \varrho_E \cdot \varrho_T = \Omega(\Delta)$, where ϱ_T is the approximation on the time-complexity by algorithm $\mathcal{A}$. We then show that for Algorithm 1, $\varrho_E = O(\Delta(G))$.

Lemma 9: Algorithm 1 is $\varrho_E = O(\Delta(G))$-approximation for energy cost.

Proof: For proof, please refer to [3]. ■

Consequently, we know that Algorithm 1 is asymptotically optimum if we want to optimize the time-complexity, message-complexity and energy-cost-complexity simultaneously. On the other hand, the minimum energy data-collection based on minimum energy path tree (MEPT) has delay that is at most $O(\Delta^4)$ times of the optimum.

Lemma 10: Data collection using MEPT is $\varrho_E = O(\Delta(G)^4)$-approximation for time complexity.

Proof: For proof, please refer to [3]. ■

VI. Data Aggregation

We consider the case when the data aggregation is distributive. In other words, given any node v and its set of children nodes $u_1, u_2, \cdots, u_d$, where d is the number of children nodes of v in a data aggregation tree, the data produced by node v has size same as the size of each of the individual node. Typical examples of such aggregation are min, max, *average*, or *variance*. In data aggregation, if one node send information twice, it can always save the first transmission. Hence, the data aggregation should be done using a tree.

A. Message, Energy, and Time Complexity

Message Complexity: The total message complexity for data aggregation using any tree T is $\Theta(n)$, where n is the number of nodes of the network. The lower bound on the message complexity n is obvious since every node v needs send at least once. The upper bound is also n because we can do the data aggregation using any spanning tree and every node only needs to send once.

Energy Complexity: For distributive aggregation, it seems that we need use some data aggregation tree that is energy efficient since each node needs to send at least once. So the main question now is to construct a tree such that the total cost of all links are minimized. This clearly is the minimum spanning tree, where the link cost of any link uv is the energy cost of sending a unit amount of data over the link uv, which can be computed in polynomial time.

Time Complexity: We will show that the time complexity for such kind of data aggregation is of the order $\Theta(H+\Delta(G))$, where H is the height of the BFS tree rooted at the sink node v_0. Notice that for a BFS tree for a graph G, its height H is $\Theta(D(G))$, thus the time complexity is of the order of $\Theta(D+\Delta(G))$ too. Algorithm 2 illustrates our method.

Theorem 11: Data aggregation can be done in $\Theta(D+\Delta)$ time with n messages.

Proof: For proof, please refer to [3]. ■

If there are more than one aggregation functions, we can deliver the messages one by one. We call this as sequential aggregation.

Corollary 12: k sequential data aggregations can be done in $O(D+\Delta+k)$ time with kn messages.

B. Complexity Tradeoffs

Again, we may want to design a data aggregation method that has constant approximation ratios for message complexity, time complexity, and energy complexity. First, aggregation based on MST (that is energy optimum for aggregation) is not efficient for time complexity.

Lemma 13: The minimum energy data aggregation based on MST is $\varrho_T = \Omega(\min(\frac{n}{\Delta}, \sqrt{n\Delta}))$-approximation for time complexity. On the other hand, $\varrho_T = O(\frac{n}{\Delta})$.

Proof: For proof, please refer to [3]. ■

Observe that our method (Algorithm 2) has constant ratio for both message complexity and time complexity. However, it is not always energy efficient due to the following lemma.

Lemma 14: Algorithm 2 is $\varrho_E = O(\Delta(G))$-approximation for energy cost.

Proof: For proof, please refer to [3]. ■

Although our method is not energy efficient in the worst case (with approximation ratio up to $\Theta(\Delta)$), we show that it is the best we can do if we want to achieve $\Theta(1)$ ratio in delay. Again, given a data aggregation method $\mathcal{A}$, let ϱ_E, ϱ_T and ϱ_M be the approximation ratios of $\mathcal{A}$ over all networks with n nodes and maximum degree Δ. We prove the following lemma.

Algorithm 2 Efficient Data Aggregation Using CDS

Input: A CDS **C** with bounded degree **d**, a distributive function f and corresponding function h.

for each dominator node v_i **do**
  For the set of dominatee nodes of the node v_i, we build a minimum spanning tree (MST) rooted at v_i, where the link weight is the energy cost for supporting the link communication. The data elements from all these dominatee nodes are then *aggregated* to the corresponding dominator node v_i along the minimum spanning tree of these dominatee nodes. In other words, any node v_k will compute $h(f(A_k), x_{k,1}, x_{k,2}, \cdots, x_{k,d_k})$ where $x_{k,j}$, for $j \in [1, d_k]$, is the aggregated value node v_k received from its jth child in the minimum spanning tree and d_k is the number of children of node v_k in the MST of all dominatee nodes of v_i. Notice that this aggregation can be done in time-slots $\Theta(\Delta(G))$.
Now we only consider the dominator nodes and the breadth-first-search spanning tree $T_{\mathbf{C}}$ of nodes in CDS rooted at the sink v_0.
for $t = 1$ to H **do**
  for each node $v_i \in V_{\mathbf{C}}$ **do**
    If node v_i has received aggregated data from all its children nodes in $T_{\mathbf{C}}$, it sends the aggregated data (using its own data and all aggregated data from its children) to its parent node in round t.

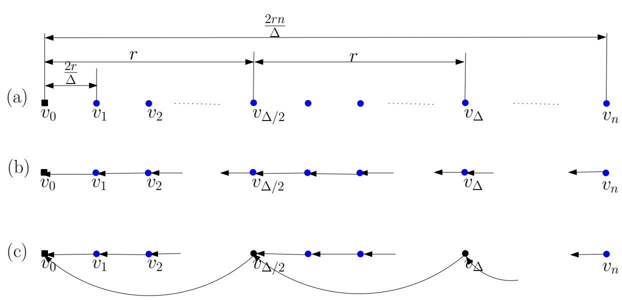

Fig. 2. Example: (a) the line network with $n+1$ nodes; (b) the minimum energy data aggregation tree; (c) the tree $T_{\mathbf{C}}$.

Lemma 15: For any data aggregation algorithm $\mathcal{A}$, there are graphs of n nodes with maximum degree Δ, such that $\varrho_T \cdot \varrho_E = \Omega(\Delta)$.

Proof: For proof, please refer to [3]. ■

C. Minimizing the maximum energy consumption by a node

We assume that all nodes have the same amount of data to be aggregated and the aggregation function f satisfies that $f(a_1, a_2, \cdots, a_p)$ has size as the maximum size of a_i, *i.e.*, the aggregated data can be represented by a single packet if each of the input data items is represented by a single packet. In this subsection, we then study how to design efficient method for minimizing the maximum energy consumption for data aggregation. We first assume that the energy consumption for sending a single data packet over all links $v_i v_j \in G$ is the same (say E) or within a small constant factor μ of each other (*i.e.*, $E(v_i v_j) \le \mu E(v_p, v_q)$ for all links $v_i v_j$ and $v_p v_q$). Observe that this is often the case in practice. We also assume that every node has P original data packets to be aggregated. For a data aggregation tree T and an internal node v_i, the energy cost is $(E_r \cdot d_{v_i}(T) + E) \cdot P$, where E_r is the energy cost for receiving a data packet, and E is the energy cost for sending a packet. To minimize the maximum energy cost of nodes v_i in T, it is equivalent to minimize $d_{v_i}(T)$, *i.e.*, the maximum node degree of T. Then it is easy to show the following lemma.

Lemma 16: Minimizing the maximum energy consumption by all nodes for data aggregation is equivalent of finding a spanning tree T with the minimum maximum node degree in T.

Finding a spanning tree T in a general graph with minimum degree is NP-hard problem. Recently, Blin *et al.* [2] proposed a distributed method that can find a spanning tree with maximum node degree at most $OPTree(G) + 1$, where $OPTree(G)$ is the smallest maximum node degree of all spanning trees for G. In this paper, we assumed that only a subset $V \subseteq U$ of nodes have data to be aggregated. Thus, we need a Steiner tree T that minimizes the maximum node degree if a single spanning tree T is needed for data aggregation. A polynomial time algorithm that produces a Steiner tree T (spanning a subset V of nodes in a general graph G) with the maximum degree at most $OPTree(G, V) + 1$ is presented in [6].

Lemma 17: If a single data aggregation tree T is required for aggregating data items from a subset V of sensor nodes, we can construct a tree T in polynomial time such that the maximum energy consumed by any node in T is at most $\mu \cdot (1 + \frac{1}{OPTree(G,V)})$ times of the optimum.

In addition to only minimizing the maximum energy cost by a node, in practice, we often need to minimize the total energy cost while the maximum energy cost by a node v_i is at most a value $E_s(v_i)$. We first show that this problem is NP-hard. Notice that for a data aggregation tree T, and a node v_i in T we know that the energy cost of node v_i is $E_r \cdot d_{v_i}(T) + E$ when we need aggregate a single packet from each of the nodes in V. Then requiring that the total energy cost of node v_i is at most a value $E_s(v_i)$ is equivalent as $E_r \cdot d_{v_i}(T) + E \le E_s(v_i)$, *i.e.*, the node degree of v_i in T, $d_{v_i}(T) \le \frac{E_s(v_i) - E}{E_r}$. Thus, an upper bound on energy consumption is equivalent as specifying an upper bound $B(v_i) = \frac{E_s(v_i) - E}{E_r}$ on the node degree of v_i in the spanning tree. Then for all spanning trees meeting this degree requirement for all nodes $v_i \in V$, we will find a spanning tree with the minimum total energy cost. Given a wireless network G, and a degree requirement specified by a vector B, let $MCost(G, B)$ be the minimum cost of all the spanning trees in G that satisfy the degree requirement B. Based on a recent result by [15], we can prove the following theorem.

Theorem 18: Given a degree upper bound $B(v_i)$ for each node v_i, there is a polynomial time algorithm that can find a data aggregation tree T such that the degree $d_{v_i}(T)$ of node v_i in T satisfies that $d_{v_i}(T) \le B(v_i) + 1$, and the energy cost of T is at most $MCost(G, B)$.

VII. Related Work

Most existing convergecast methods [1, 7, 16] are based on a tree structure and with minimum either energy or data latency as the objective. For example, [16] first constructs a tree using greedy approach and then allocates DSSS or FHSS codes for its nodes to achieve collision-free, while [1, 7] uses TDMA to avoid collisions. In [1], the authors did *not* give any theoretical tradeoffs between energy cost and latency. Zhang and Huang [18] proposed a hop-distance based temporal coordination heuristic for adding transmission delays to avoid collisions. They studied the effectiveness of packet aggregation and duplication mechanisms with such convergecast framework. Kennelman and Kowalski [10] proposed a randomized distributed algorithm for convergecast that has the expected running time $O(\log n)$ and uses $O(n \log n)$ times of minimum energy in the worst case, where n is the number of nodes. They also showed the lower bound of running time of any algorithm in an arbitrary network is $\Omega(\log n)$. However, they assume that all nodes can dynamically adjust its transmission power from 0 to any arbitrary value and a data message by a node can contain *all* data it has collected from other nodes. In [4], Chu *et al.* studied how to provide approximate and bounded-loss data collection in sensor networks instead of accurate data. Their method used replicated dynamic probabilistic models to minimize communication from sensor nodes to the base station.

To significantly reduce communication cost in sensor networks, in-network aggregation has been studied and implemented. In TAG (Tiny AGgregation service) [11], besides the basic aggregation types (such as *count*, *min*, *max*, *sum*, *average*) provided by SQL, five groups of possible sensor aggregates are summarized: distributive aggregates (*e.g.*, *count*, *min*, *max*, *sum*), algebraic aggregates (*e.g.*, *average*), holistic aggregates (*e.g.*, *median*), unique aggregates (*e.g.*, *count distinct*), and content-sensitive aggregates (*e.g.*, *fixed-width histograms* and *wavelets*). Notice that the first two groups aggregates are very easy to achieve by a tree-based method. To overcome the severe robustness problems of the tree approaches [11, 12, 17], multipath routing for in-network aggregation has been proposed [5, 14]. Then recently Manjhi *et al.* [13] combined the advantages of the tree and multi-path approaches by running them simultaneously in different regions of the network. In [8], Kashyap *et al.* studied a randomized (gossip-based) scheme using which all the nodes in a complete overlay network can compute the common aggregates of $\min$, $\max$, sum, $average$, and $rank$ of their values using $O(n \log \log n)$ messages within $O(\log n \log \log n)$ rounds of communication. Kempe *et al.* [9] earlier presented a gossip-based method which can get the average in $O(\log n)$ rounds with $O(n \log n)$ messages.

VIII. Conclusion

In this paper, we study the time complexity, message complexity, and energy complexity of data collection and data aggregation problems in wireless sensor networks. We first present efficient algorithms that achieve asymptotically optimal message complexity, energy complexity, and time complexity. We then study the lower bound of the complexities for these problems and show that our method achieves the best possible tradeoffs among these three metrics. There are still a number of interesting questions left for future research. One is to design efficient algorithms when each node will produce a data stream. The second challenge is what is the best algorithm when we do not require that the found data item to be precise, *i.e.*, we allow certain relative errors, or additive errors on the found answer.

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