

# Joint QoS-aware Admission Control, Channel Assignment, and Power Allocation for Cognitive Radio Cellular Networks

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**Abstract**—In cognitive radio cellular networks (CogCells), primary users (PUs) rarely utilize all the assigned frequency bands at a certain time and a location. The spectral inefficiency caused by the spectrum holes motivated cognitive radio technology (CR) that presents unlicensed secondary users (SUs) an opportunity for using spectrum holes. CR makes the SUs to find and use the spectrum holes without interrupting the operation of PUs. The SUs are allowed to access the channel licensed to the PUs which consist of primary transmitters (PTs) and primary receivers (PRs) when the interference to the PRs is less than acceptable value (i.e., predefined system threshold), and the quality of service (QoS) required by PTs are also guaranteed. According to different levels of QoS required by SUs, the network operator can achieve different secondary revenues by providing different QoS levels to SUs. Due to the high density, the mobility of SUs, the interference limitation at PRs and the QoS requirements from PTs, not all SUs can be supported. The problem we investigated in this paper is to select the maximum subset of SUs to maximize the total secondary revenue of the CogCell, meanwhile the QoS requirements from both PTs and admitted SUs must be guaranteed. Moreover, the interference caused by the admitted SUs and the PTs at the PRs (due to access the same channel) has to be less than the predefined system threshold. In this paper, we formulate such a joint QoS-aware admission control, channel assignment, and power allocation scheme as a non-linear NP-hard optimization problem. This is a very challenging problem and the NP-hardness has been shown in the literature even for the single-channel scenario. In this paper, we propose a new polynomial-time joint QoS-aware admission control, channel assignment and power allocation scheme which has a  $O(\frac{1}{\log n_p^r + \log n_w})$  approximation guarantee, e.g., the total secondary revenue achieved by our algorithm is at least  $\Omega(\frac{1}{\log n_p^r + \log n_w})$  of the optimum, where  $n_p^r$  is the number of PRs and  $n_w$  is the number of available channels in the CogCell. Note that Our algorithm also significantly improves the current best known solution with a  $O(\frac{1}{n_p^r})$  approximation guarantee for the single-channel scenario [11]. In this paper, we also propose a greedy heuristic approximation algorithm and an exact solution. The simulation results show that the approximation algorithms we proposed can achieve significantly higher secondary revenue than the currently best known approximation approach for this problem, an extension of the minimal SINR removal algorithm in [15]. Indeed, quite surprisingly, the simulation results also demonstrate that the secondary revenue achieved by our approximation approaches is very close to the optimum in practice, specially for the  $O(\frac{1}{\log n_p^r + \log n_w})$ -approximation algorithm.

## I. INTRODUCTION

In cognitive radio cellular networks (CogCells), primary users (PUs) rarely utilize all the assigned frequency bands at

a certain time and a location which has been shown in the literature. The spectral inefficiency caused by the spectrum holes motivated cognitive radio technology (CR) that present unlicensed Secondary Users (SUs) an opportunity for using spectrum holes. CR makes the SUs to find and use the spectrum holes without interrupting the operation of PUs. Moreover, CR shows great promise to enhance the spectrum utilization efficiency [5]. A CogCell normally consists of one base station (BS), several SUs, several PUs which can be cataloged into primary transmitters (PTs) and primary receivers (PRs) and several channels which can be used in the system. Spectrum sharing by SUs may significantly affect the quality of service (QoS) of the CogCell, e.g., increasing the interference power received at PRs, decreasing the signal-interference-plus-noise-ratio (SINR) of PTs if the SUs share the same channel with the PUs and so on. However, the behaviors of SUs will not affect the QoS of the PUs which are accessing different channels with the SUs. Consequently, the admission control scheme at BS plays a very important role in the CogCell. Moreover, the admitted SUs are also supposed to be provided certain QoS by adapting its own transmission power. Therefore, the joint admission control, channel assignment, and power allocation schemes become even more interesting and challenging in the CogCell.

Joint admission control, channel assignment, and power allocation problem for maximization of secondary revenue in the CogCell has not been explored well so far. Most of work only focused on the single-channel scenario. Islam et al. [7] investigated the distributed scheme in the CogCell with one BS equipped with multiple antennas, one PT and one PR. In [12] and [13], Xing et al. proposed a distribute constrained power control algorithm under consideration of the CogCell with one PR, and several SUs with separative secondary receivers, which based on a game-theory approach. In [15], Zhang et al. proposed a minimal SINR removal algorithm (MSRA) for the CogCell with one PU. The NP-hardness of this problem had also been showed in [15]. An power control scheme had been proposed in [16] for the CogCell under a strong assumption that all SUs are admitted to access the channel to BS. In [11], Xiang et al. introduced three QoS-aware admission and power control schemes which included an exact solution based on dynamic programming, a greedy heuristic algorithm and a minimal SINR removal algorithm which is a simple extension

of MSRA from [15].

In this paper, we investigate the multiple-channel scenario in CogCells and formulate such a joint QoS-aware admission control, channel assignment, and power allocation scheme as a non-linear NP-hard optimization problem. We propose a new polynomial-time joint QoS-aware admission control, channel assignment and power allocation scheme which has a  $O(\frac{1}{\log n_p^r + \log n_w})$  approximation guarantee, e.g., the total secondary revenue achieved by our algorithm is at least  $\Omega(\frac{1}{\log n_p^r + \log n_w})$  of the optimum. Note that Our algorithm also significantly improves the current best known algorithm with a  $O(\frac{1}{n_p^r})$  approximation guarantee for the single-channel scenario [11]. In this paper, we also propose a greedy heuristic approximation algorithm and an exact solution. The simulation results show that the approximation algorithms we proposed can achieve significantly higher secondary revenue than the currently best known approximation approach for this problem, an extension of the minimal SINR removal algorithm in [15]. Indeed, quite surprisingly, the simulation results also demonstrate that the secondary revenue achieved by our approximation approaches is very close to the optimum in practice, specially for the  $O(\frac{1}{\log n_p^r + \log n_w})$ -approximation algorithm.

The rest of the paper is organized as follows. In Section II, we introduce the network model and system architecture of CogCells, the model of wireless interference, and the QoS metrics for PUs and the admitted SUs which followed by the formulation of the network operator problem in Section III. In Section IV, we present our new joint QoS-aware admission control, channel assignment, and power allocation schemes. In Section V, the superiority on the performance of our scheme is demonstrated. Finally, we conclude the paper in Section VI.

## II. SYSTEM MODEL AND ASSUMPTIONS

In this section, we will firstly introduce the network model and system architecture we employed for CogCells. Then we describe the model of wireless interferences caused by PTs and admitted SUs and the definition of SINR. Finally, we give the metrics of the QoS requirements which have to be guaranteed and provided to PUs and admitted SUs in the CogCell system.

### A. Network Model and System Architecture

In the literature, the models used in CogCells for spectrum sharing between PUs and SUs can be cataloged into two main classes. The first one is called overlay model where SUs should stop transmission on the channels they currently used once PUs are detected. Another one is underlay model in which the SUs and PUs can coexist and share the same spectrum with each other by employing Code Division Multiple Access (CDMA) as long as the interference caused by the SUs to PUs is less than the predefined system threshold [2], [10], [11]. In this paper, we concentrate on the underlay model. In a CogCell, the system consists of one BS, several PUs and SUs, and several channels which can be used in the system. However, any admitted SU (or PUs) is only allowed to use at most one channel at any time due to the single radio interface

which is equipped to SUs (or PUs). In general, the modes of PUs can be divided into two status: transmitting and receiving according to the data situation which the PUs are dealing with. Without loss of generality, the PUs in the CogCell can be represented by the PTs where the PUs are transmitting (or planning to transmit) data on some channels, and the PRs where the PUs stay in the receiving mode. Another motivation of such a division on PUs is to guarantee different QoS requirements which will be described later. Figure 1 shows the system model of the CogCell we employed in this paper.

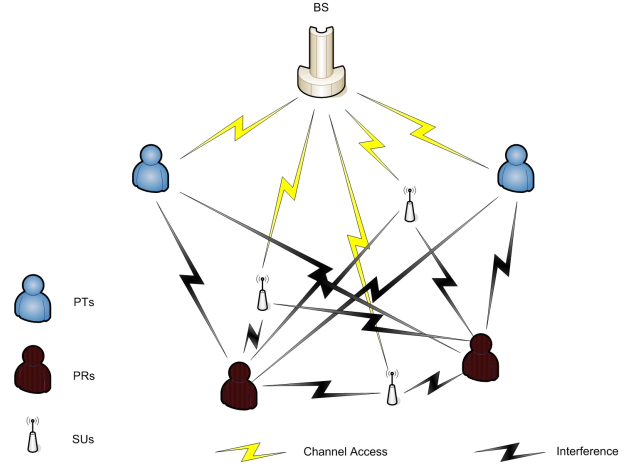


Fig. 1. An example of the CogCell.

To simplify our presentation and clarify the novelty and significance of our work later, we use the similar notations as ones used in [11], but for the scenario with multiple channels. Let  $N_s, N_p^t, N_p^r$  and  $N_w$  denote the sets of SUs, PTs, PRs, and the available channels respectively. The number of SUs, PTs, PRs, and channels can be denoted by  $n_s, n_p^t, n_p^r$  and  $n_w$  respectively, where  $n_s = |N_s|, n_p^t = |N_p^t|, n_p^r = |N_p^r|$ , and  $n_w = |N_w|$ . Moreover, let  $N_p^t(w), N_p^r(w), n_p^t(w), n_p^r(w)$  denote the sets of PTs and PRs which are accessing on channel  $w$  and the cardinalities of the corresponding sets respectively. In a CogCell system, the operator of the BS can receive the *secondary revenue* from the payments made by the admitted SUs in service and will try to maximize the potential *secondary revenue*. Assume that each SU  $i \in N_s$  will pay  $r_i$  to the operator if the QoS demand in term of minimum DTR  $\lambda_i^{sq}$  is satisfied. However, each SU  $i$  will also generate an interference  $\tau_{ij}$  to PR  $j$  if SU  $i$  is allowed to access the exactly same channel as PR  $j$ . Since the PUs have higher privilege than the SUs in the CogCell, the interference caused by all admitted SUs and all PTs to any PR  $j$  can not exceed the predefined system threshold  $\Gamma_j$ . In order to maximize the secondary revenue, a proper subset of the SUs (e.g., admitted SUs) need to be selected to access the channels to the BS with acceptable interference at each PRs. Moreover, the system also need to grantee and provide the QoS to PTs and admitted SUs according to the individual requirements. The detailed QoS requirements from PTs and admitted SUs will be introduced in following sections. For easy reading, we list all notations

TABLE I  
NOTATIONS

Term	Definition
CogCell	cognitive radio cellular networks
BS	base station
PU	primary user
SU	secondary user
PTs	PTs which are in transmitting mode
PRs	PRs which are in receiving mode
$N_s$	the set of SUs
$N_p^t$	the set of PTs
$N_p^t(w)$	the set of PTs on channel $w$
$N_p^r$	the set of PRs
$N_p^r(w)$	the set of PRs on channel $w$
$n_s$	the number of SUs
$n_p^t$	the number of PTs
$n_p^t(w)$	the number of PTs on channel $w$
$n_p^r$	the number of PRs
$n_p^r(w)$	the number of PRs on channel $w$
$P_i^s(w)$	the transmission power of SU $i$ on channel $w$
$P_k^p(w)$	the transmission power of PU $k$ on channel $w$
$P_{max}^s$	the maximum transmission power at SUs
$P_{max}^p$	the maximum transmission power at PTs
$r_i$	the revenue from SU $i$
$r_{min}$	the minimum revenue from SUs
$r_{max}$	the maximum revenue from SUs
$\tau_{ij}^w$	the interference to PR $j$ by SU $i$ on channel $w$
$\zeta_{kj}^w$	the interference to PR $j$ by PT $k$ on channel $w$
$\Gamma_j^w$	the interference threshold at PR $j$ on channel $w$
$h_{ij}^{sb}(w)$	the power attenuation from SU $i$ to BS
$h_{ij}^{sp}(w)$	the power attenuation from SU $i$ to PR $j$
$h_{kj}^{pp}(w)$	the power attenuation from PT $k$ to PR $j$
$h_k^{sp}(w)$	the power attenuation from PT $k$ to BS
$d_{ij}^{sb}$	the distance from SU $i$ to BS
$d_{ij}^{sp}$	the distance from SU $i$ to PR $j$
$d_{kj}^{pp}$	the distance from PT $k$ to PR $j$
$d_k^{sp}$	the distance from PT $k$ to BS
$I_s(w)$	the interference received at BS from all permitted SUs
$I_p(w)$	the interference received at BS caused by the PTs
SINR	signal-to-interference-plus-noise ratio [8]
$\xi_i^s(w)$	the SINR of SU $i$ measured at BS on channel $w$
$\xi_k^p(w)$	the SINR of PT $k$ measured at BS on channel $w$
$\xi_i^{min,s}$	the minimum SINR required by SU $i$
$\xi_k^{min,p}$	the minimum SINR required by PT $k$
$\lambda_i^s(w)$	the DTR achieved by SU $i$
$\lambda_k^p(w)$	the DTR achieved by PT $k$
$\lambda_i^{min,s}$	the minimum DTR required by SU $i$
$\lambda_k^{min,p}$	the minimum DTR required by PT $k$
$G_i^s(w)$	the antenna gain of SU $i$
$G_k^{pt}(w)$	the antenna gain of PT $k$
$G_j^{pr}(w)$	the antenna gain of PR $j$
$\alpha$	the path fading factor
$T_j^I(w)$	the interference power accumulated at PR $j$ on channel $w$
DTR	data transmission rate

and symbols used in this paper in Table I.

### B. Wireless Transmission and Interference Model

When SUs and PTs transmit with the same channel of PRs, PRs will receive interference power from all these admitted SUs and PTs. Different types of interference models have been studied in the literature, which includes physical interference model (PhyIM) [4], [11], [15], [16], fixed protocol interferences model (fPrIM) [9], RTS/CTS model (RTS-CTS)

[1], transmitter interference model (TxIM) [14]. In this paper, we adopt the PhyIM. Let  $\tau_{i,j}^w$  and  $\zeta_{k,j}^w$  denote the interference power received by PR  $j$  due to transmission from SU  $i \in N_s$  and PT  $k \in N_p^t$  on channel  $w$  respectively. According to the PhyIM,  $\tau_{i,j}^w$  and  $\zeta_{k,j}^w$  can be expressed as follows.

$$\tau_{i,j}^w = h_{ij}^{sp}(w)P_i^s(w), \forall i \in N_s, \forall j \in N_p^r \quad (1)$$

and

$$\zeta_{k,j}^w = h_{kj}^{pp}(w)P_k^p(w), \forall k \in N_p^t, \forall j \in N_p^r \quad (2)$$

where  $P_i^s(w)$ ,  $P_k^p(w)$ ,  $h_{ij}^{sp}(w)$  and  $h_{kj}^{pp}(w)$  denote the transmission powers on channel  $w$  at SU  $i$  and PT  $k$ , and the power attenuation from SU  $i$ , PT  $k$  to the PR  $j$  respectively. From the PhyIM,  $h_{ij}^{sp}(w)$  and  $h_{kj}^{pp}(w)$  can be calculated as follows.

$$h_{ij}^{sp}(w) = \frac{G_i^s(w)G_j^{pr}(w)}{(d_{ij}^{sp})^\alpha}, \forall i \in N_s, \forall j \in N_p^r \quad (3)$$

and

$$h_{kj}^{pp}(w) = \frac{G_k^{pt}(w)G_j^{pr}(w)}{(d_{kj}^{pp})^\alpha}, \forall k \in N_p^t, \forall j \in N_p^r \quad (4)$$

where  $G_i^s(w)$ ,  $G_k^{pt}(w)$ ,  $G_j^{pr}(w)$ ,  $d_{ij}^{sp}$ ,  $d_{kj}^{pp}$ , and  $\alpha$  denote the antenna gains of SU  $i$ , PT  $k$  and PR  $j$  on channel  $w$ , the distances from the SU  $i$  and PT  $k$  to the PR  $j$ , and the path fading factor respectively.

Consequently, the interference power  $T_j^I(w)$  accumulated at PR  $j$  on channel  $w$  due to the transmission from all admitted SUs and PTs with exactly same channel  $w$  can be formulated as follows.

$$\begin{aligned} T_j^I(w) &= \sum_{i=1}^{n_s} \tau_{ij}^w x_{iw} + \sum_{k=1}^{n_p^t} \zeta_{kj}^w \\ &= \sum_{i=1}^{n_s} \frac{G_i^s(w)G_j^{pr}(w)P_i^s(w)x_i}{(d_{ij}^{sp})^\alpha} + \sum_{k=1}^{n_p^t} \frac{G_k^{pt}(w)G_j^{pr}(w)P_k^p(w)}{(d_{kj}^{pp})^\alpha} \\ &\leq \Gamma_j^w, \forall j \in N_p^r \end{aligned} \quad (5)$$

where  $x_{iw}$  is a binary variable where  $x_{iw} = 1$  indicates SU  $i$  is admitted to transmit to the BS on channel  $w$ , otherwise SU  $i$  is forbidden to transmit on channel  $w$  in the CogCell. Due to the privileges of PUs and coexistence regulation in underlay model [11], [15],  $T_j^I(w)$  can not exceed the predefined system threshold  $\Gamma_j^w$  on channel  $w$  at PR  $j$ . Note that the interference model used in [11], [15] is not reliable since the interference limitation at PR  $j$  does not take account into the accumulated interference from PTs. Without loss of the generality, we always assume that

$$\sum_{k=1}^{n_p^t} \zeta_{kj}^w \leq \sum_{k=1}^{n_p^t} \frac{G_k^{pt}(w)G_j^{pr}(w)P_{max}^p(w)}{(d_{kj}^{pp})^\alpha} \leq \Gamma_j^w, \forall j \in N_p^r. \quad (6)$$

### C. SINR Definition

According to the description of PhyIM, the SINR of PT  $k$  on channel  $w$  can be expressed from a trivial extension of single channel scenario in [11] as follows.

$$\xi_k^p(w) = \frac{h_k^{pb}(w)P_k^p(w)}{N_0 + I_s(w) + I_p(w) - h_k^{pb}(w)P_k^p(w)}, \forall k \quad (7)$$

where  $h_k^{pb}(w)$  and  $P_k^p(w)$  denote the power attenuation to the BS and the transmission power of PT  $k$  on channel  $w$  respectively. Moreover,  $N_0$  present the background noise received at the BS,  $I_s(w)$  and  $I_p(w)$  present the interferences received from all admitted SUs and PTs on channel  $w$  respectively. According to the definition of SINR,  $I_s(w)$  and  $I_p(w)$  can be defined as follows.

$$I_s(w) = \sum_{i=1}^{n_s} h_i^{sb}(w)P_i^s(w)x_{iw} \quad (8)$$

$$I_p(w) = \sum_{k=1}^{n_p^t} h_k^{pb}(w)P_k^p(w) \quad (9)$$

where  $h_i^{sb}(w)$  and  $h_k^{pb}(w)$  denote the power attenuation on the channel  $w$  from SU  $i$  and PT  $k$  to the BS respectively.

By the definition of SINR, we can also have

$$h_i^{sb}(w) = \frac{G_i^s(w)G^b(w)}{(d_i^{sb})^\alpha}, \forall i \in N_s \quad (10)$$

and

$$h_k^{sp}(w) = \frac{G_k^{pt}(w)G^b(w)}{(d_k^{pb})^\alpha}, \forall k \in N_p^t \quad (11)$$

where  $d_i^{sb}$ ,  $d_k^{pb}$ ,  $G_k^{pt}(w)$  and  $G^b(w)$  denote the distances from SU  $i$  and PT  $k$  to the BS and the antenna gains of PT  $k$  and the BS on the channel  $w$  respectively.

Similarly as Equation 7, the SINR of the admitted SU  $i$  can be given by

$$\xi_i^s(w) = \frac{h_i^{sb}(w)P_i^s(w)}{N_0 + I_s(w) + I_p(w) - h_i^{sb}(w)P_i^s(w)} \quad (12)$$

$$\forall i \in N_s \wedge (x_{iw} = 1),$$

where  $\wedge$  denotes logical AND operator.

### D. QoS Requirements

The main QoS measurement we considered here is the data transmission rate (DTR), which is the same metric on the QoS used in [11], [15]. According to Shannon' channel capacity formulation, the maximum DTR  $\lambda$  can be estimated by

$$\lambda = B \log_2(1 + \xi) \quad (13)$$

where  $B$  is the channel bandwidth and  $\xi$  is the SINR.

To guarantee the minimum DTR  $\lambda_i^{min,s}$  required by the admitted SU  $i$  and the minimum DTR  $\lambda_k^{min,p}$  required by PT  $k$ , it is equivalent to guarantee the minimum SINRs  $\xi_i^{min,s}$  for SU  $i$  and  $\xi_k^{min,p}$  for PT  $k$  according to Equation 13.

$$\xi_i^s(w) \geq \xi_i^{min,s} = 2^{\frac{\lambda_i^{min,s}}{B}} - 1, \forall i \in N_s \wedge (x_{iw} = 1) \quad (14)$$

and

$$\xi_k^p(w) \geq \xi_k^{min,p} = 2^{\frac{\lambda_k^{min,p}}{B}} - 1, \forall k \in N_p^t. \quad (15)$$

## III. THE PROBLEM FORMULATION

The problem we investigated in this paper is to control the transmission powers of SUs and PTs, and select a subset of SUs to assign proper channels to the admitted SUs in order to maximize total secondary revenue (Equation 16) which can be achieved at the operator of the CogCell, where  $r_i$  denotes the revenue paid by the admitted SU  $i$ . Meanwhile, the interference limitation on PRs, the QoS requirements for both PTs and admitted SUs have to be guaranteed. We formulate such a joint QoS-aware admission control, channel assignment, and power allocation as a non-linear NP-hard problem as follows.

$$\text{argmax}_{x_{iw}, P_i^s(w), P_k^p(w)} \text{max} \sum_{i=1}^{n_s} \sum_{w=1}^{n_w} r_i x_{iw} \quad (16)$$

subject to:

$$\sum_{i=1}^{n_s} \sum_{w=1}^{n_w} \tau_{ij}^w + \sum_{k=1}^{n_p^t} \sum_{w=1}^{n_w} \zeta_{kj}^w \leq \Gamma_j^w, \forall j \in N_p^r, \forall w \in N_w \quad (17)$$

$$\sum_{w=1}^{n_w} x_{iw} \leq 1, \forall i \in N_s \quad (18)$$

$$x_{iw} \in \{0, 1\}, \forall i \in N_s, \forall w \in N_w \quad (19)$$

$$\xi_k^p(w) \geq \xi_k^{min,p}, \forall k \in N_p^t \quad (20)$$

$$\xi_i^s(w) \geq \xi_i^{min,s}, \forall i \in N_s \wedge (x_{iw} = 1) \quad (21)$$

$$\xi_i^s(w) = 0, \forall i \in N_s \wedge (x_{iw} = 0) \quad (22)$$

$$P_i^s(w) = 0, \forall i \in N_s \wedge (x_{iw} = 0) \quad (23)$$

$$P_i^s(w) \in [0, P_{max}^s], \forall i \in N_s \quad (24)$$

$$P_k^p(w) \in [0, P_{max}^p], \forall k \in N_p^t \quad (25)$$

where the first constraint (Equation 17) says that the interference to any PR  $j$  due to the transmissions from all admitted SUs and PTs with same channel with PR  $j$  can not exceed the predefined system interference threshold. The constraint (Equation 18) ensures that only one channel can be used by any admitted SUs due to only one radio interface which is equipped to any SUs in the CogCell. The binary variable  $x_{iw}$  (Equation 19) is used to indicate whether SU  $i$  is admitted to access the channel  $w$  to the BS, e.g.,  $x_{iw} = 1$  means that SU  $i$  is admitted on channel  $w$ , and  $x_{iw} = 0$  indicates that SU  $i$  is not allowed to use channel  $w$  in the CogCell system. The following two constraints (Equations 20, 21) describe the QoS requirements in which PTs and all admitted SUs have to be satisfied. The next two constraints (Equation 22, 23) clarify the situation when SU  $i$  is forbidden to access the channels in the system. The last two constraints (Equations 24, 25) give the power limitations on SUs and PTs respectively.

#### IV. JOINT QoS-AWARE ADMISSION CONTROL, CHANNEL ASSIGNMENT, AND POWER ALLOCATION

In this section, we present three new joint QoS-aware admission control, channel assignment, and power allocation algorithms which includes a greedy heuristic  $O(\frac{1}{n_p^r \cdot n_w})$  approximation scheme (GHAS), a  $O(\frac{1}{\log n_p^r + \log n_w})$  approximation algorithm (Fast-AA), and an exact solution. Our Fast-AA scheme can be also used to significantly improve the current best known approximation algorithm with a  $O(\frac{1}{n_p^r})$  approximation guarantee for the single-channel scenario [11].

##### A. Preliminary Power Control Scheme for PTs

In this section, we propose a new polynomial-time preliminary power control scheme for PTs which guarantee the QoS requirements for all PTs  $k \in N_p^t(w)$  in term of DTR or SINR (see Equations 15 and 20). We also prove that the proposed power control scheme for PTs is optimal since it can produce best opportunity for the operator to achieve the maximum secondary revenue in the CogCell system. This scheme will be used in our approximation algorithms described later.

According to the Equations 7 and 9, we can derive

$$I_s(w) = \frac{h_k^{pb}(w)P_k^p(w)}{\xi_k^p(w)} - N_0 - \sum_{\substack{1 \leq u \leq n_p^t \\ u \neq k}} h_u^{pb}(w)P_u^p(w), \forall k \quad (26)$$

To distinguish the different  $I_s(w)$  allowed by different PT on channel  $w$  in Equation 26 to guarantee the QoS requirement at PT  $k$ , we use  $I_s^k(w)$  denote the interference gain allowed by PT  $k \in N_p^t(w)$  on channel  $w$ .

In order to maximize secondary revenue at operator, loosely speak that we need to maximize the number of potential SUs to be admitted to the system due to small diversity of the payment for the service which made by the SUs in the CogCell. This is equivalent to maximize the interference upper bound  $I_s(w)$  allowed by SUs meanwhile the QoS requirements from all PTs and PRs should be also guaranteed.

From Equation 26, it is easy to see that  $I_s^k(w)$  can be enlarged by decreasing  $\xi_k^p(w)$  or increasing  $P_k^p(w)$ . However, the increment of  $P_k^p$  will decrease  $I_s^u(w)$  according to Equation 26, where  $k \in N_p^t(w)$ ,  $u \in N_p^t(w)$  and  $k \neq u$ .

Since all PT  $k \in N_p^t$  are licensed users in the CogCell, the QoS required by any PT  $k \in N_p^t(w)$  must be guaranteed. Consequently, how to choose optimal power for each PT  $k \in N_p^t(w)$  sharing the same channel  $w$  in order to maximize the interference upper bound  $I_s(w)$  can be formulated to the following optimization problem.

$$ArgP_k^p(w): \forall k \in N_p^t(w) MaxMin(I_s^k(w)) \quad (27)$$

subject to:

$$\xi_k^p(w) \geq \xi_k^{min,p}, \forall k \in N_p^t(w) \quad (28)$$

$$P_k^p(w) \in [0, P_{max}^p], \forall k \in N_p^t(w) \quad (29)$$

$$\sum_{i=1}^{n_s} \tau_{ij}^w + \sum_{k=1}^{n_p} \zeta_{kj}^w \leq \Gamma_j^w, \forall j \in N_p^r(w) \quad (30)$$

The objective (Equation 27) is to maximize the minimum interference upper bound allowed by PT  $k \in N_p^t(w)$ . The first constraint (Equation 28) describe the QoS requirements for each PT  $k \in N_p^t(w)$  which have to be guaranteed. The second constraint (Equation 29) give the power limitation for PTs. The last constraint (Equation 30) is to guarantee the QoS requirements from PRs.

Let  $I_s^k(\xi_k^{min,p}, w)$  denotes the interference allowed by PT  $k \in N_p^t(w)$  with guaranteed QoS in term of SINR  $\xi_k^{min,p}$  on channel  $w$ . A necessary condition to achieve the optimal solution for the problem mentioned in Equation 27 is

$$I_s^1(\xi_1^{min,p}, w) = I_s^2(\xi_2^{min,p}, w) \dots = I_s^{n_p^t(w)}(\xi_{n_p^t(w)}^{min,p}, w) \quad (31)$$

Combining with Equation 26, this necessary condition (Equation 31) is equivalent to the following constraint.

$$P_k^p(w) = \frac{(1 + \frac{1}{\xi_1^{min,p}})h_1^{pb}(w)}{(1 + \frac{1}{\xi_k^{min,p}})h_k^{pb}(w)}P_1^p(w), \forall k \in N_p^t(w) \quad (32)$$

Without loss of generality, we assume that  $P_\beta^p(w) \geq P_k^p(w)$  holds for  $\forall k \in N_p^t(w)$  according to Equation 32, where  $\beta \in N_p^t(w)$ . Without loss of generality, we assume that  $P_k^p$  is a constant integer for  $\forall k \in N_p^t(w)$ , e.g.,  $300mW$ .

According to Equation 26, the upper bound of  $I_s^\beta(w)$  can be enlarged if we increase  $P_\beta^p(w)$ . However, on the other hand, increase of powers at PTs can reduce the number of SUs which can be admitted to access the channel to BS in the CogCell due to the interference constraints at PRs (Equation 30) and the accumulated interference from PTs (by Equation 2). Combining the upper bound of power limitation  $P_{max}^p$  for all PTs by Equation 32, the power of PT  $k \in N_p^t(w)$  can be allocated as follows.

$$P_k^p(w) = \frac{(1 + \frac{1}{\xi_\beta^{min,p}})h_\beta^{pb}(w)}{(1 + \frac{1}{\xi_k^{min,p}})h_k^{pb}(w)}P_\beta^p(w) \quad (33)$$

where  $k \in N_p^t(w)$  and  $P_\beta^p(w) \leq P_{max}^p$ . To simplify our presentation, we can also compute a minimum power  $P_{min}^\beta(w)$  in order to guarantee all QoS from PTs when none of SUs are admitted to access the channel  $w$  in the CogCell, where

$$P_{min}^\beta(w) = \frac{N_0}{\frac{h_\beta^{pb}(w)}{\xi_\beta^{min,p}} - \sum_{\substack{1 \leq k \leq n_p^t(w) \\ k \neq \beta}} \frac{(1 + \frac{1}{\xi_\beta^{min,p}})h_\beta^{pb}(w)}{(1 + \frac{1}{\xi_k^{min,p}})}} \quad (34)$$

The optimal value of  $P_\beta^p(w)$  will be determined in the later sections.

Therefore, in order to guarantee the QoS from all PTs and PRs of the CogCell, the maximal interference  $I_s^{max}(w)$  on channel  $w$  can be bounded by:

$$\leq \frac{h_\beta^{pb}(w)P_\beta^p(w)}{\xi_\beta^{min,p}} - N_0 - \sum_{\substack{1 \leq k \leq n_p^t(w) \\ k \neq \beta}} \frac{(1 + \frac{1}{\xi_\beta^{min,p}})h_\beta^{pb}(w)}{(1 + \frac{1}{\xi_k^{min,p}})}P_\beta^p(w) \quad (35)$$

### B. Post Procedure of Power Control for SUs

Different with PTs, a SU  $i \in N_s$  does not have to be admitted to the CogCell system due to the interference constraints on PUs or unsatisfied QoS requirements and so on. As mentioned before, selection of an optimal subset of SUs to maximize the total secondary revenue of the CogCell is a very challenging problem and the NP-hardness has been proved in the literature even for the single-channel scenario. In this section, we investigate the problem how to set the power for the admitted SUs which are sharing the same channel in order to allow more SUs to be admitted to share this channel in the CogCell. The new power control scheme we proposed here for SUs will be used in our approximation algorithms as a post procedure to further increase the opportunity for other SUs which can be admitted to the system.

For the instant, we assume that we already have a subset  $N'_s(w) \subseteq N_s$  which are admitted to access the channel  $w$  in the CogCell. Similar as the power control approach for PTs, our scheme also guarantees that the QoS requirements for all admitted SUs  $i \in N'_s(w)$  in term of DTR (see Equations 14 and 21). After execution of the preliminary power control scheme for PTs, we can allocate the powers for admitted SUs. For the instant, we assume that we know the exact value of  $P_\beta^p(w)$  which introduced in Section IV-A.

According to the Equations 12, 9 and 33, we can derive

$$\begin{aligned} I_s(w) &= h_i^{sb}(w)P_i^s(w)\left(1 + \frac{1}{\xi_i^s(w)}\right) - N_0 - I_p(w) \\ &= h_i^{sb}(w)P_i^s(w)\left(1 + \frac{1}{\xi_i^s(w)}\right) - N_0 \\ &\quad - \sum_{k=1}^{n_p^t(w)} h_k^{pb}(w) \frac{\left(1 + \frac{1}{\xi_\beta^{min,p}}\right)h_\beta^{pb}(w)}{\left(1 + \frac{1}{\xi_k^{min,p}}\right)h_k^{pb}(w)} P_\beta^p(w) \end{aligned} \quad (36)$$

To distinguish the different upper bound  $I_s(w)$  allowed by different admitted SUs  $i \in N'_s(w)$ , we use  $I_s^{s,i}(w)$  to denote the interference gain allowed by the admitted SU  $i$ .

Similarly as Section IV-A, how to set optimal power for each admitted SU  $i \in N'_s(w)$  to maximize the upper bound of the interference constraint  $I_s$  can be formulated to the following optimization problem.

$$Arg_{P_i^s(w): \forall i \in N'_s(w)} MaxMin(I_s^{s,i}(w)) \quad (37)$$

subject to:

$$\xi_i^s(w) \geq \xi_i^{min,s}, \forall i \in N'_s(w) \quad (38)$$

$$P_i^s(w) \in [0, P_{max}^s], \forall i \in N'_s(w) \quad (39)$$

$$\sum_{i=1}^{|N'_s(w)|} \tau_{ij}^w + \sum_{k=1}^{n_p^t(w)} \zeta_{kj}^w \leq \Gamma_j^w, \forall j \in N_p^r(w) \quad (40)$$

Let  $I_s^{s,i}(\xi_i^{min,s}, w)$  denotes the upper bound of the interference allowed by the active SU  $i \in N'_s(w)$  with guaranteed QoS in term of SINR  $\xi_i^{min,s}$ . A necessary condition to achieve the optimal solution for 37 is

$$I_s^{s,1}(\xi_1^{min,s}, w) = I_s^{s,2}(\xi_2^{min,s}, w), \dots, = I_s^{|N'_s(w)|}(\xi_{|N'_s(w)|}^{min,s}, w)$$

Combining with Equation 36, it can be rewrote as follows.

$$P_i^s(w) = \frac{\left(1 + \frac{1}{\xi_1^{min,s}}\right)h_1^{sb}(w)}{\left(1 + \frac{1}{\xi_i^{min,s}}\right)h_i^{sb}(w)} P_1^s(w), \forall i \in N'_s(w) \quad (41)$$

Without loss of generality, we assume that  $P_\pi^s(w) \geq P_i^s(w)$  holds for  $\forall i \in N'_s(w)$  by Equation 41, where  $\pi \in N'_s(w)$ .

Combining the upper bound of power limitation  $P_{max}^s$  for all admitted SUs with the power allocation scheme by Equation 41, the power of SU  $i \in N'_s(w)$  can be allocated as follows.

$$P_i^s(w) = \frac{\left(1 + \frac{1}{\xi_\pi^{min,s}}\right)h_\pi^{sb}(w)}{\left(1 + \frac{1}{\xi_i^{min,s}}\right)h_i^{sb}(w)} P_\pi^s(w) \quad (42)$$

where  $i \in N'_s(w)$  and  $P_\pi^s(w) \leq P_{max}^s$ . Given  $P_\pi^s(w)$ , the optimal power for other SUs can be allocated according to the power control scheme by 42. The optimal value of  $P_\pi^s(w)$  will be determined in our approximation algorithms later.

### C. A Greedy Heuristic Algorithm

In this section, we propose a greedy heuristic approximation algorithm (GHAA) to solve the joint QoS-aware admission control, channel assignment, and power allocation problem.

In our GHAA scheme, the selection of admitted SUs  $i \in N_s$  and assignment of the channels  $w$  for the admitted SUs are crucially based on a preference function  $e_i(w)$  described as follows.

$$e_i(w) = \frac{r_i}{\sum_{j \in N_p^r(w)} \left( \sum_{i \in N'_s(w)} \tau_{ij}^w - (\Gamma_j^w - \sum_{k \in N_p^t(w)} \zeta_{kj}^w) \right) \tau_{ij}^w} \quad (43)$$

which is an extension from the single-channel scheme in [11].

Having  $e_i(w)$ , the selection of admitted SUs can be done in a greedy manner. In each iteration of GHAA, one unique SU  $i$  with largest  $e_i(w)$  will be admitted to access the channel  $w$  in the CogCell, where  $w \in N_w$ . Combining the power control schemes in Sections IV-A and IV-B, the details of GHAA are deferred in Algorithm 1. The GHAA scheme we proposed here is much simple and easy to be implemented however it is easy to see that GHAA trivially leads a  $O(\frac{1}{n_w n_p^r})$ -approximation ratio if all admitted SUs have the largest interference component on selected channel  $w$  at a PR  $j \in N_p^r(w)$ . Motivated by the disadvantage of GHAA, we propose another approximation algorithm with much better approximation guarantee in Section IV-D.

### D. A $O(\frac{1}{\log n_p^r + \log n_w})$ -approximation algorithm

In this section, we present a new polynomial-time joint QoS-aware admission control, channel assignment, and power allocation scheme in which our algorithm can guarantee that the secondary revenue achieved at the operator is at least  $\Omega(\frac{1}{\log n_p^r + \log n_w})$  of the optimum meanwhile the QoS requirements from PTs, PRs and the admitted SUs are also guaranteed. This scheme can be also used to significantly improve the current best known approximation algorithm with a  $O(\frac{1}{n_p^r})$  approximation guarantee for the single-channel scenario [11]. Our algorithm bases on the frame work for

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**Algorithm 1** Greedy Heuristic Approximation Algorithm (GHAA).

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**Input:**  $N_w, N_p^t(w), N_p^r(w), N_s, P_{max}^s, P_{max}^p, r_i, \Gamma_j^w, h_i^{sb}(w), h_{ij}^{sp}(w), h_{kj}^{pp}(w), h_k^{sp}(w), d_i^{sb}, d_{ij}^{sp}, d_{kj}^{pp}, d_k^{sp}, G_i^s(w), G_k^{pt}(w), G_j^{ppr}(w), \alpha$ .

**Output:**  $N'_s(w)$  (sets of admitted SUs  $\forall w \in N_w$ ),  $P_k^p(w)$ ,  $P_i^s(w)$  and  $revenue_{max}$ .

- 1:  $revenues_{max}^w = 0; N_{max}^s = \emptyset; P_k^{p'}(w) = 0; P_i^{s'}(w) = 0;$
  - 2: find  $\beta \in N_p^t(w)$  according to 32;
  - 3: compute  $P_{min}^\beta(w)$  according to 34;
  - 4: **for**  $power_p^w = P_{min}^\beta(w) + 1$  to  $P_{max}^p$  **do**
  - 5:  $P_\beta^p(w) = power_p^w;$
  - 6:  $N'_s(w) = \emptyset;$
  - 7: set  $P_k^p(w)$  according to 33;
  - 8: find  $\pi \in N'_s(w)$  according to 41;
  - 9: **for**  $power_s^w = 1$  to  $P_{max}^s$  **do**
  - 10: set  $P_i^s(w)$  according to 42;
  - 11: select best SU  $i \in (N_s - N'_s)$  on channel  $w$  according to the preference  $e_i(w)$  43;
  - 12: **if** constraint 49 for SU  $i$  holds **then**
  - 13:  $N'_s(w) = N'_s(w) + \{i\};$
  - 14:  $revenues_{max}^w = revenues_{max}^w + r_i;$
  - 15:  $N_{max}^s(w) = N'_s(w);$
  - 16:  $P_k^{p'}(w) = P_k^p(w);$
  - 17: re-allocate the power for SU  $i \in N'_s(w)$  to minimize  $\tau_{ij}^w$  by 48;
  - 18:  $P_i^{s'}(w) = P_i^s(w);$
  - 19:  $N_{max}^s(w) = N'_s(w);$
  - 20:  $N'_s(w) = N_{max}^s(w);$
  - 21:  $P_i^s(w) = P_i^{s'}(w);$
  - 22:  $P_k^p(w) = P_k^{p'}(w);$
  - 23:  $revenue_{max} = \sum_{w=1}^{n_w} revenue_{max}^w;$
- 

set covering problems which called  $\rho$ -approximation subset oblivious algorithm in [3], and a new observation that  $n_w n_p^r$ -dimension (multiple multi-dimension) bin packing is a  $O(n_w n_p^r)$ -approximation subset oblivious.

1) *Preliminaries:* To simplify our presentation and the analysis of our algorithm later, we briefly introduce the frame work for the set covering problems which called  $\rho$ -approximation subset oblivious algorithm in [3]. A new observation from [3] will be used as a sub-procedure in our main algorithm in Section IV-D2.

Given a set  $I$  of  $d$ -dimensional items, the  $i$ -th corresponding to a  $d$ -tuple  $(t_i^1, t_i^2, \dots, t_i^d)$ , that must be packed into the smallest number of unit-size bins, corresponding to the  $d$ -tuple  $(1, \dots, 1)$ . Given an instant  $I$ , let  $opt(I)$  denote the value of the optimal solution for  $I$ . This problem can be formulated as the following general set covering problem, in which a set  $I$  of items has to be covered by configurations from the collection  $\mathcal{C} \subseteq 2^I$ , where each configuration  $C \in \mathcal{C}$  corresponds to a set

of items that can be packed into a bin:

$$\min \left\{ \sum_{C \in \mathcal{C}} y_C : \sum_{i \in C} y_C \geq 1 (i \in I), y_C \in \{0, 1\} (C \in \mathcal{C}) \right\} \quad (44)$$

Since the collection  $\mathcal{C}$  is exponentially large for the given application item set  $I$ , an approximation algorithm (or LP relaxation of 44) can be very useful for such an application.

The dual of this LP (Equation 44) is given by

$$\max \left\{ \sum_{i \in I} w_i : \sum_{i \in C} w_i \leq 1 (C \in \mathcal{C}), w_i \geq 0 (i \in I) \right\} \quad (45)$$

Note that the separation problem for the dual is the following knapsack-type problem: given weights  $w_i$  on each item  $i$ , find a feasible configuration in which the total weight of items does not exceed 1. In the literature, it has been show that:

*Theorem 1:* If there exists a Polynomial-Time Approximation Scheme (PTAS) for the separation problem for 45, that is given  $w_i \in \mathbb{R}_+^{|I|}$  solve  $\max_{C \in \mathcal{C}} \sum_{i \in C} w_i$ , then there exists a PTAS for the LP relaxation of 44.

Based on Theorem 1, an approximation solution of the set covering problem 44 has been constructed in [3], which consists the following steps, where  $\delta > 0$  is a parameter whose value can be specified later.

**Step 1:** Solve the LP relaxation of 44, possibly approximately in case  $\mathcal{C}$  is exponentially large in the input size. Let  $y^*$  be the (near)-optimal solution of the LP relaxation and  $z^* = \sum_{C \in \mathcal{C}} y_C^*$  be its value. Let  $C_1, C_2, \dots, C_m \in \mathcal{C}$  be the configurations associated with the nonzero components of  $y^*$ ;

**Step 2:** Define the binary vector  $y^r$  starting with  $y_C^* = 0$  for  $C \in \mathcal{C}$  and  $S = I$  (i.e., all items are uncovered) and then repeating the following for  $\lceil \delta z^* / (1 - \sigma\psi/2) \rceil$  iterations: select the configuration  $C' \in \{C_1, C_2, \dots, C_m\}$  such that  $\Phi(S - C')$  is minimum and let  $y_{C'}^r = 1$  and  $S = S - C'$ , where  $\sigma$  is a small parameter such that  $\sigma\psi < 1$  to be specified later. For an arbitrary set of items  $S$ ,  $\Phi(S) = \ln(\sum_{j=1}^d e^{\sigma \sum_{i \in S} w_i^j})$ ;

**Step 3:** Consider the set of items  $S \subseteq I$  that are not covered by  $y_r$ , namely  $i \in S$  if and only if  $\sum_{i \in C} y_C^r = 0$ , and the associated optimization problem for the residual instance

$$\min \left\{ \sum_{C \in \mathcal{C}} y_C : \sum_{i \in C} y_C \geq 1 (i \in S), y_C \in \{0, 1\} (C \in \mathcal{C}) \right\}. \quad (46)$$

Apply some approximation algorithm to the problem 46 yielding solution  $y^a$ ;

**Step 4:** Return the solution  $y^h = y^r + y^a$ .

For the abbreviation, we denote this approach by  $\text{SETCOVER}(I, w)$ , where  $w$  is the weight vector for  $\forall i \in I$ .

A crucial notation used in [3] is called the  $\rho$ -approximation subset oblivious algorithm which defined as follows.

*Definition 2:* A  $\rho$ -approximation algorithm for problem 44 is called subset oblivious if, for any fixed  $\varepsilon > 0$ , there exist constraints  $d, \psi, \varpi$  (possibly depending on  $\varepsilon$ ) such that, for every instance  $I$  of 44, there exists vectors  $w^1, w^2, \dots, w^d \in \mathbb{R}^{|I|}$  with the following properties: (i)  $\sum_{i \in C} w_i^d \leq \psi$ , for each  $C \in \mathcal{C}$  and  $j = 1, 2, \dots, d$ ; (ii)  $opt(I) \geq \max_{j=1}^d \sum_{i \in I} w_i^j$ ;

(iii)  $\text{appr}(S) \leq \rho \max_{j=1}^d \sum_{i \in S} w_i^j + \varepsilon \text{opt}(I) + \varpi$ , for each  $S \subseteq I$ .

The following theorem has been shown in [3].

*Theorem 3:* A  $\rho$ -approximation subset oblivious problem can be solved by procedure SETCOVER( $I, w$ ) with  $O(\log \rho)$  approximation guarantee.

In what follows, we show a new observation which makes us to achieve a new approximation algorithm with a better approximation guarantee shown in Section IV-D2.

*Theorem 4:*  $\iota$   $d$ -dimensional (multiple multi-dimensional) bin packing is a  $O(\iota \cdot d)$ -approximation subset oblivious algorithm.

*Proof:* It stated in [3] that there exists a polynomial-time  $O(d)$ -approximation subset-oblivious algorithm for  $d$ -dimensional bin packing. Consequently,  $\iota$   $d$ -dimensional bin packing is a  $O(\iota \cdot d)$ -approximation subset oblivious algorithm by iteration of  $d$ -dimensional bin packing at most  $\iota$  times. More details on analogous analysis can be found in Lemma 3 [3]. ■

*Theorem 5:* The cost of the final heuristic solution for  $\iota$   $d$ -dimensional bin packing produced by procedure SETCOVER( $I, w$ ) with  $\delta = \ln d \cdot \iota$ ,  $\sigma = (2\varepsilon / \ln d \cdot \iota) / (\psi + \psi\varepsilon / \ln d \cdot \iota)$  and  $\psi = 1$  is at most

$$(\ln d \cdot \iota + 1 + 2\varepsilon) \text{opt}(I) + \delta + \frac{(2 \ln d \cdot \iota)(1 + \varepsilon / \ln d)}{(2\varepsilon / \ln d \cdot \iota)} + 1, \quad (47)$$

i.e., this is a deterministic  $O(\log d + \log \iota)$ -approximation algorithm for  $\iota$   $d$ -dimensional bin packing (e.g., problem 44).

*Proof:* It directly follows due to Theorem 4 and Theorem 3. ■

2) *The Fast-AA Algorithm:* In this section, we present our main approximation algorithm (FastAA) with  $O(\frac{1}{\log n_p^r + \log n_w})$  approximation ratio.

The main idea of our algorithm is that we first convert a variety of our problem to a  $n_w n_p^r$ -dimensional (multiple multi-dimensional) bin packing problem according to the preliminary power allocation for PTs in Section IV-A, where each feasibly potential admitted SUs are the items, all PRs and the available channel  $n_w$  are the  $n_w n_p^r$ -dimensional bin. Given the power control scheme for PTs in Section IV-A (for the instant, we assume that we know the power allocation  $P_\beta^p(w)$ ), the interference limitation (Equation 5) at each PR  $j \in N_p^r(w)$  can be recomputed based on Equation 33. Therefore, the interference allowed at PR  $j$  due to the transmission from the potential admitted SUs can be bounded as follows.

$$\begin{aligned} \sum_{i=1}^{n_s} \tau_{ij}^w x_{iw} &\leq \Gamma_j^w - \sum_{k=1}^{n_p^t(w)} \zeta_{kj}^w \\ &= \Gamma_j^w - \sum_{k=1}^{n_p^t(w)} \frac{G_k^{pt}(w) G_j^{pr}(w) P_k^p(w)}{(d_{kj}^{pp})^\alpha} \\ &= \Gamma_j^w - \sum_{k=1}^{n_p^t(w)} \frac{G_k^{pt}(w) G_j^{pr}(w) (1 + \frac{1}{\xi_{\beta}^{min,p}}) h_\beta^{pb}(w) P_\beta^p(w)}{(d_{kj}^{pp})^\alpha (1 + \frac{1}{\xi_k^{min,p}}) h_k^{pb}(w)} \\ \forall j &\in N_p^r(w) \end{aligned} \quad (48)$$

Note that a feasibly potential active SU  $i$  refers that

$$\begin{aligned} \tau_{ij}^w &\leq \Gamma_j^w - \sum_{k=1}^{n_p^t(w)} \frac{G_k^{pt}(w) G_j^{pr}(w) (1 + \frac{1}{\xi_{\beta}^{min,p}}) h_\beta^{pb}(w) P_\beta^p(w)}{(d_{kj}^{pp})^\alpha (1 + \frac{1}{\xi_k^{min,p}}) h_k^{pb}(w)}, \\ \forall j &\in N_p^r(w), \exists w \in N_w. \end{aligned} \quad (49)$$

Consequently, all PRs on particular channel  $w$  corresponds to a  $n_p^r(w)$ -tuple unit-size bin  $(1, \dots, 1)$ , and a feasibly potential admitted SU  $i$  corresponds to a  $n_p^r(w)$ -tuple item  $(w_i^1, w_i^2, \dots, w_i^{n_p^r(w)})$ , where

$$w_i^j = \frac{\tau_{ij}^w}{\Gamma_j^w - \sum_{k=1}^{n_p^t(w)} \frac{G_k^{pt}(w) G_j^{pr}(w) (1 + \frac{1}{\xi_{\beta}^{min,p}}) h_\beta^{pb}(w) P_\beta^p(w)}{(d_{kj}^{pp})^\alpha (1 + \frac{1}{\xi_k^{min,p}}) h_k^{pb}(w)}} \quad (50)$$

Note that  $w_i^j$  can be different for different channels. After execution of the heuristic algorithm for  $n_w n_p^r$ -dimensional bin packing produced by procedure SETCOVER( $N_s, w$ ), combining with the secondary revenue  $r_i$  on each potential admitted SU  $i$ , we select one bin for each channel  $w$  which can achieve maximal secondary revenue among all bins we configured for channel  $w$ . Moreover, we only allow the SUs in this selected bin to access corresponding channel  $w$ . Then we perform the SU power control scheme (see Section IV-B) for all admitted SUs with same channel. In case that the selected bin still has space left (i.e., interference limitation) after re-allocation of the power for each admitted SUs, we can admit more SUs to be active based on the preference function mentioned in 43 if only if the QoS of the new admitted SUs and other already admitted SUs can be guaranteed. The details are illustrated in Algorithm 2.

*Theorem 6:* Our FastAA scheme can guarantee that the secondary revenue achieved at the operator are at least  $\Omega(\frac{1}{\log n_p^r + \log n_w})$  of the optimum meanwhile the QoS required by all PTs, PRs and the admitted SUs are satisfied.

*Proof:* The QoS requirements of all PTs, PRs and the admitted SUs can be satisfied due the construction of  $N_s'$ , e.g., the SU  $i$  can not be included in  $N_s'$  if the QoS are not satisfied. Without consideration of the different revenues at each SUs of the CogCell (e.g., if the revenues are same at different SUs), our scheme can achieve at least  $\Omega(\frac{1}{\log n_p^r + \log n_w})$  of the optimum due to Theorems 3, 4 and 5. Taking account into the factor of different revenues at different SUs, the achievement of our scheme only reduces a constant fact  $\frac{r_{max}}{r_{min}}$ , where  $r_{max}, r_{min}$  denote the maximal revenue and the minimal revenue among all SUs respectively. It completes the proof. ■

## E. An Exact Solution

Due to the enormous memory requirement by the dynamic programming approaches for the problem we formulated in Section III, we employ the standard branch-and-bound approach in [6] to solve our problem. The crucial property of branch-and-bound techniques is an intelligent enumeration of the solution space for the optimization problems. It can divide



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**Algorithm 2** Fast Approximation Algorithm (FastAA).

---

**Input:**  $N_w, N_p^t(w), N_p^r(w), N_s, P_{max}^s, P_{max}^p, r_i, \Gamma_j^w, h_i^{sb}(w), h_{ij}^{sp}(w), h_{kj}^{pp}(w), h_k^{sp}(w), d_i^{sb}, d_{ij}^{sp}, d_{kj}^{pp}, d_k^{sp}, G_i^s(w), G_k^{pt}(w), G_j^{pp}(w), \alpha.$

**Output:**  $N_s'(w)$  (sets of admitted SUs  $\forall w \in N_w$ ),  $P_k^p(w), P_i^s(w)$  and  $revenue_{max}$ .

- 1:  $revenues_{max}^w = 0; N_{max}^s = \emptyset; P_k^{p'}(w) = 0; P_i^{s'}(w) = 0;$
  - 2: find  $\beta \in N_p^t(w)$  according to 32;
  - 3: compute  $P_{min}^\beta(w)$  according to 34;
  - 4: **for**  $power_p^w = P_{min}^\beta(w) + 1$  to  $P_{max}^p$  **do**
  - 5:    $P_\beta^p(w) = power_p^w;$
  - 6:    $N_s'(w) = N_s;$
  - 7:   set  $P_k^p(w)$  according to 33;
  - 8:   find  $\pi \in N_s'(w)$  according to 41;
  - 9:   **for**  $power_s^w = 1$  to  $P_{max}^s$  **do**
  - 10:     set  $P_i^s(w)$  according to 42;
  - 11:     **if** constraint 49 for SU  $i$  does not hold **then**
  - 12:        $N_s'(w) = N_s'(w) - \{i\}$
  - 13:       compute  $w_i^j$  according to 50;
  - 14:       execute SETCOVER( $N_s', w$ );
  - 15:       select one bin  $\mathfrak{B}$  with maximal secondary revenue achieved on channel  $w$ ;
  - 16:     **if**  $\sum_{i \in \mathfrak{B}} r_i > revenues_{max}^w$  **then**
  - 17:        $revenues_{max}^w = \sum_{i \in \mathfrak{B}} r_i;$
  - 18:        $N_{max}^s(w) = \mathfrak{B};$
  - 19:        $N_s'(w) = \mathfrak{B};$
  - 20:        $P_k^{p'}(w) = P_k^p(w);$
  - 21:       re-allocate the power for SU  $i \in N_s'(w)$  to minimize  $\tau_{ij}^w$  by 48;
  - 22:        $P_i^{s'}(w) = P_i^s(w);$
  - 23:       **if** the bin  $\mathfrak{B}$  still has some space **then**
  - 24:         add more SU ( $\forall w : i \in N_s - N_s'(w)$  to  $N_s'(w)$ ) according to the preference  $e_i(w)$  43;
  - 25:        $N_{max}^s(w) = N_s'(w);$
  - 26:      $N_s'(w) = N_{max}^s(w);$
  - 27:      $P_i^s(w) = P_i^{s'}(w);$
  - 28:      $P_k^p(w) = P_k^{p'}(w);$
  - 29:  $revenue_{max} = \sum_{w=1}^{w=n_w} revenue_{max}^w;$
- 

the original problem into several decomposed subproblems and calculate these subproblems in parallel which significantly curtails the computational burden. According to the methodologies described in [6], the original optimization problem we presented in Section III can be transferred to the following problem with exactly same constraints as the original ones.

$$arg_{x_{iw}, P_i^s(w), P_k^p(w)} max(\mathbb{L}(\mathfrak{X}, \mathfrak{y}, \mathfrak{z})) \quad (51)$$

where

$$\begin{aligned} \mathbb{L}(\mathfrak{X}, \mathfrak{y}, \mathfrak{z}) = & \sum_{i=1}^{n_s} \sum_{w=1}^{n_w} r_i x_{iw} - \sum_{i=1}^{n_s} \mathfrak{y}_i \left( \sum_{w=1}^{n_w} x_{iw} - 1 \right) \\ & - \sum_{w=1}^{n_w} \mathfrak{z}_w \left( \tau_{ij}^w + \sum_{k=1}^{n_p^t} \zeta_{kj}^w - \Gamma_j^w \right), \forall j \in N_p^r, \end{aligned} \quad (52)$$

TABLE II  
DIR AND REVENUE

DTR (kbps)	16	32	64	128	256	512
Revenue	1	1.5	2	2.5	3	3.5

where  $\eta_i$  and  $\mathfrak{z}_w$  are dual variables. It is easy to see that optimal solution of the dual problem (Eq. 51) is an upper bound of optimum of the original problem for arbitrary nonnegative  $\eta_i$  and  $\mathfrak{z}_w$ . However, to achieve an optimal solution in terms of a tight upper bound, the optimum dual variables have to be chose such that  $\mathbb{L}(\mathfrak{X}, \mathfrak{y}, \mathfrak{z})$  (Eq. 52) is minimized. Due to the space constraint, the standard branch-and-bound procedures thus are omitted. The reader can refer [6] for the details.

## V. SIMULATION RESULTS

In this section, we evaluate our schemes (GHAA and FastAA) with the extension of the minimal SINR removal algorithm (MSRA) in [15], [11]. Meanwhile, we also compare the secondary revenue our approximation algorithms achieved with the optimal solution.

### A. Environment Setup

The CogCell in our simulation is randomly generated within a [1000m, 1000m] area. For the fairness, we locate the BS in the center with coordinate (500, 500). Without loss of generality, we set all antenna gains of SUs, PTs, PRs, and the BS as 1. Moreover,  $P_{max}^p$  and  $P_{max}^s$  are allocated by 300mW and 260mW respectively. The channel bandwidth  $B$  is set as 5MHz. The background noise is given by  $1.0 \times 10^{-14}$ . The number of channels are randomly generated from 3 to 5, which forms the available channel set in the CogCell. Moreover, the channels accessed by PTs and PRs are randomly generated from the available channel set.

The setting of the revenues with corresponding DTR required by SUs can be found in Table II, where the revenue for higher DTR will be larger.

The DTR demands of PTs and SUs are randomly generated from the choices listed in Table II.

### B. Performance Evaluation

We vary the numbers of SUs, PRs, and the interference thresholds of PRs to evaluate the performance of our approximation schemes (GHAA and FastAA) with MSRA. Each data used in our evaluation is the average number computed over 10 different CogCell. Note also that the condition (Equation 6) holds for all example we investigated.

Figure 2 shows the results for the scenario with various number of SUs. In this example, both the numbers of PTs and PRs are fixed at 5, the number of SUs varies from 5 to 15. The interference threshold for all PRs is set at  $10^{-10}W$ . It is clear that the performance on secondary revenue achieved by our schemes is better than MSRA scheme, specially for the situation when larger number of SUs are employed in the CogCell.

Figure 3 shows the results for the scenario with various number of PRs. In this example, the number of PTs is fixed

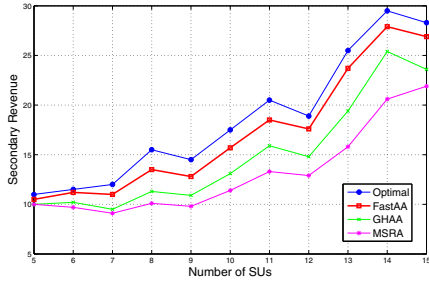


Fig. 2. Secondary revenue in terms of the number of SUs

at 5, and the number of SUs is fixed at 10, the number of PRs varies from 5 to 15. The interference threshold for all PRs is set at  $10^{-10}W$ . The result shows that a higher secondary revenue can be achieved when small number of PRs are employed in the CogCell. It also shows that the secondary revenues achieved by our schemes GHAA and FastAA are higher than MSRA scheme.

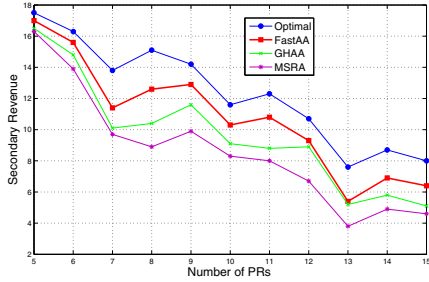


Fig. 3. Secondary revenue in terms of the number of PRs

Figure 4 shows the results for the scenario with various interference threshold of PRs. In this example, we assume that the interference threshold for all PRs are same. The number of PTs is fixed at 5, and the number of SUs is fixed at 10, the interference threshold of PRs varies from  $-105dBW$  to  $-95dBW$ . It is clear that a higher secondary revenue can be achieved when larger interference threshold of PRs is set at the system.

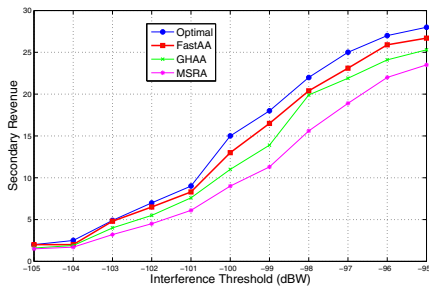


Fig. 4. Secondary revenue in terms of the interference threshold of PRs

Note also that in all examples here, the secondary revenue achieved by our FastAA scheme is very close to the optimum,

which also demonstrates the superiority of our methods.

## VI. CONCLUSION

In this paper, we investigate the multiple-channel scenario in CogCells and formulate the joint QoS-aware admission control, channel assignment, and power allocation as a non-linear NP-hard optimization problem. We propose a new polynomial-time joint QoS-aware admission control, channel assignment and power allocation scheme which has a  $O(\frac{1}{\log n_r^p + \log n_{rv}})$  approximation guarantee. Note that Our algorithm also significantly improves the current best known algorithm with a  $O(\frac{1}{n_r^p})$  approximation guarantee for the single-channel scenario [11]. In this paper, we also propose a greedy heuristic approximation algorithm and an exact solution. The simulation results show that the approximation algorithms we proposed can achieve significantly higher secondary revenue than the currently best known approximation approach for this problem, an extension of the minimal SINR removal algorithm in [15], [11]. Indeed, quite surprisingly, the simulation results also demonstrate that the secondary revenue achieved by our approximation approaches is very close to the optimum in practice, specially for the FastAA algorithm.

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