An Analytical Study of Subdividing Hexagon-Clustered WSN for Power Saving

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Abstract—Hexagon is an ideal shape for clustering sensor networks, for it can seamlessly divide clustered areas, and is the largest regular polygon (in terms of the number of sides) that has this property. In this paper, we analyze the benefit of subdividing a hexagonal cluster for the purpose of reducing the overall power consumption in the cluster. Assuming a *spatial Poisson distribution* of sensor nodes in the cluster, we propose a subdivision scheme, and perform a comprehensive analytical estimate of power savings brought about by the subdivision. The analytical results show that subdivision will yield considerable saving in overall power consumption of the cluster, and the saving is heavily dependent on the nodes' transmission range and their deployment density. The limit on the depth of subdivision is also analyzed.

Keywords: Clustered WSN, Energy efficiency, Hierarchy, Hexagonal, spatial Poisson distribution, Wireless Sensor Networks.

1 Introduction

A Wireless Sensor Network (WSN) is composed of a large number of sensor nodes, and a few (at least one) base stations. The sensors are deployed in various physical environments mainly for collecting physical world data. The data are transmitted to, or gathered by, the base station for aggregation, analysis, and processing. The communication among nodes is done via wireless means. Therefore all nodes are equipped with radio transceivers/receivers. WSNs have very promising prospect in many applications, such as environment monitoring, traffic monitoring, target tracking, and fire detection, to name a few.

Numerous different models of WSN have been proposed. To support scalability and improve efficiency, it is a common technique to group nodes into disjoint and mostly non-overlapping clusters. In the past, many clustering schemes have been proposed, citing advantages on various metrics such as convergence rate, cluster stability, cluster overlapping, location-awareness, and most importantly, prolonged network lifetime [1, 4, 5, 8, 13, 14, 15, 19]. In a cluster-based, hierarchical structure, there is a group of sensor nodes functioning as clusterheads (CHs) to collect data from their neighboring nodes. The data traffic can be greatly reduced by applying data aggregation at clusterheads. Cluster members have low energy consumption, as they transmit sensor data to a nearby node. The CHs form a second layer of network. The selection of CHs depends on factors such as topology of the WSN, the applications, and the optimization objectives. One invariable target is to keep the battery-operated WSN's working life as long as possible.

In this paper, we are interested in the benefit of subdividing a WSN with hexagonal clusters to the end of saving power. Different shapes have been pro-

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posed to cluster the area where WSN nodes are deployed. The circle, square, and hexagon are among the most popular ones. The circular is the "most natural" cluster because it coincides with the shape of the radio transmission range. The shortcoming of it is that clusters have to be overlapped, or there will be uncovered areas. The hexagon is an ideal shape for clustering a large terrain into adjacent, non-overlapping areas. It is the largest polygon (in terms of number of sides) that has this property (the only other polygons are triangle and square).

A typical operation of a clustered WSN is that all sensor nodes in a cluster transmit the sensed data to their clusterhead, and the clusterhead transmits the data (raw or preprocessed) to the base station. We will propose a subdivision of the original hexagonal clusters, so that the overall power consumption of the WSN for data collection/processing is greatly reduced. With the given architectural and communication models, and assuming a spatial Poisson distribution of sensor nodes, we will perform an analytical estimate of the power saving by the proposed subdivision strategy. We will analyze subdivision of various scales, and the corresponding power saving patterns. The analytical results have shown that subdivision will yield considerable saving in overall power consumption of the cluster. The deeper the subdivision, the more power can be saved.

The rest of this paper is organized as follows. In Section 2, we describe the underlying clustered WSN model for which the power-saving subdivision is proposed. In Section 3, we perform a comprehensive analysis on the overall saving of power by subdividing the hexagonal cluster into sub-clusters of various sizes. Assuming Poisson distribution of sensor nodes, we will develop analytical framework to estimate the power saving, and determine the limit on depth of subdivision. We will also examine the relationship between data aggregation rate and subdivision. We give concluding remarks and briefly discuss directions of future work in Section 4.

2 The sensor network model

2.1 Network model

Because of the wide diversity of sensor network applications, it is hard to address all requirements of a WSN in a single model. In this paper, we will adopt a rather commonplace WSN model, with the assumption that the nodes are deployed in the area of interest. The WSN comprises a large number of low power, low cost sensors, deployed in a large physical environment. The distribution of sensors is modeled by the Poisson distribution. The whole WSN is clustered into a set of hexagonal clusters. For each cluster, there is a clusterhead at the center of the hexagon, which is costlier, more powerful in computational capability, and has larger radio transmission range. The sensors' main job is to collect first-hand, raw physical data, with or without some initial processing. The clusterheads perform more intensive, more complex tasks. It is at the clusterheads that the data of the sensor network get processed in a collaborative manner. Figure 1 illustrates the basic structure of the hierarchical WSN.

In Figure 1, the sensors (represented by white circles) are densely but evenly distributed in a two-dimensional terrain. At the center of the hexagonal cluster is a clusterhead (represented by a bigger, black circle). For the purpose of energy saving, an ordinary sensor's communication capability is presumably limited, consuming as low as possible power in radio transmission. We assume it can transmit as far as to the clusterhead of its own cluster. A clusterhead is equipped with more powerful transmitter that can communicate with other clusterheads. However, again for the purpose of energy saving, we do not assume a limitless, super powerful clusterhead that can send/receive radio messages to all sensors/clusterheads in the system. Rather, the transmission range of a clusterhead is assumed to be approximately twice of a sensor node, which is just the hexagon's maximal diameter, denoted by D. In other words, the clusterhead's transmission range determines the size of the hexagonal clusters. Accordingly, we denote the maximal radius $R = D/2$.

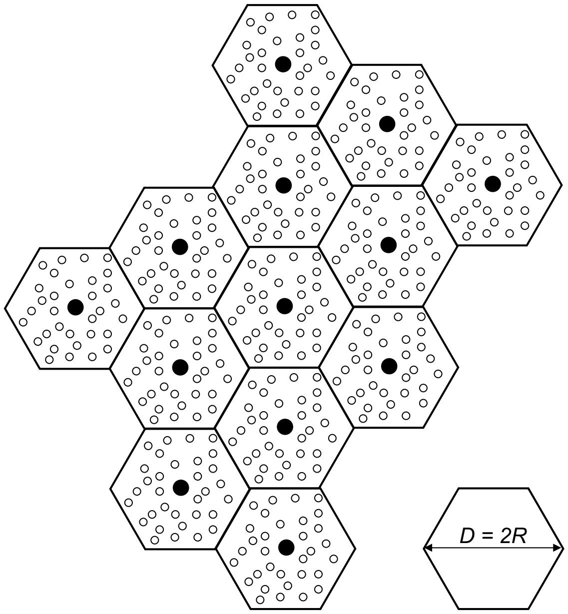

Figure 1: A hexagonal-clustered wireless sensor network. The black circles at the center are clusterheads. The maximal diameter of the hexagon is D, which is the transmission distance of the clusterhead. The distance between two clusterheads is $\frac{\sqrt{3}D}{2} = \sqrt{3}R$.

This will allow the clusterhead to be able to transmit to any neighboring clusterhead, which is of distance $\sqrt{3}R = \frac{\sqrt{3}D}{2}$.

2.2 Power consumption and communication models

Suppose the distance from a transmitting node u to a receiving node v is x. A simple and widely used model for estimating the power consumed for transmitting a unit (e.g., a packet) of message from u to v is as follows:

$$E_{u,v} = x^{\alpha} \tag{1}$$

where α is a constant larger than or equal to 2. This power consumption model has been adopted in many earlier studies [2, 11, 3, 9, 10, 17]. Some works use $E_{u,v} = x^{\alpha} + c$, where c is a constant representing the power consumed on overhead in the transmitting node. Following the model in [7], we will use Equation (1) with $\alpha = 2$ in this paper's analysis. We are interested in the scenario of one round of data collection. That is, every sensor sends its sensed data to the clusterhead. The total power consumed is then just the sum of powers consumed by all nodes transmitting to the clusterhead.

To send data to the clusterhead, each sensor uses "just enough" power to transmit, as illustrated in Figure 2 (a). According to Equation (1), the farther the node from the clusterhead, the more power it uses. Therefore in Figure 2 (a), we have $E_1 > E_2$.

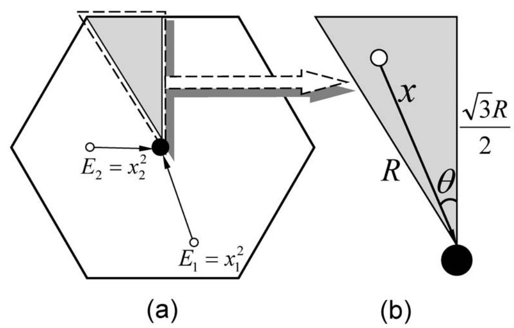

Figure 2: (a) The longer distance to clusterhead, the more power a node uses to transmit data; (b) If the distribution of nodes were uniform and dense, we could use a double definite integral $\int_0^{\frac{\pi}{6}} \int_0^{\frac{\sqrt{3}R}{2\cos\theta}} x^2 dx d\theta$ to approximate the total transmission power in the shaded triangle.

If the distribution of nodes were uniform as well as dense, we could use a double definite integral to approximate the total transmission power for all nodes in a cluster. Referring to Figure 2 (b) and using Equation (1), the power consumed by all nodes in the shaded 30°-60°-90° triangle area can be approximated by the following definite integral:

$$\int_0^{\frac{\pi}{6}} \int_0^{\frac{\sqrt{3}R}{2\cos\theta}} x^2 dx d\theta$$

where R is the maximal radius of the hexagon, which is half of the clusterhead transmission range D ($R = \frac{D}{2}$).

3 Subdivision of the cluster for power-efficiency

3.1 Spatial Poisson distribution

Throughout this paper, we model the deployment of sensor nodes with spatial Poisson distribution. Poisson distribution is a widely used model for probabilistic temporal counting (such as the arrival of customers at a bank) and spatial counting (such as the population in a randomly chosen area). The main characteristic of spatial Poisson distribution is that the larger the area, the larger the population. We assume that in the area of sensor deployment, this is the case.

Spatial Poisson distribution can be defined in a d-dimensional space $\mathbb{R}^d$, $d = 1, 2, 3$. For our purpose, we assume that the nodes in a cluster follow spatial Poisson distribution in a 1-dimensional space $\mathbb{R}$. See Figure 3 for illustration. A parameter λ (determined by observation) represents the *density* of the sensor node distribution, i.e, the average number of nodes in a unit length. The larger the λ, the more nodes in a given segment $A \subset \mathbb{R}$.

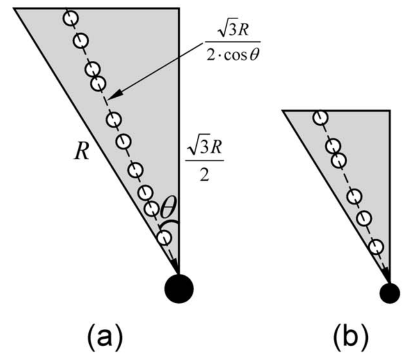

Figure 3: Spatial Poisson distribution: The radius R is larger in (a) than in (b). Therefore for a given angle θ, on the corresponding radial segment, there will be more nodes in (a) than in (b).

3.2 Energy consumption without subdivision

Let $|A|$ denote the length of segment $A \subset \mathbb{R}$. It can be shown that the expected number of nodes in A, denoted $E(N_A)$, is equal to $\lambda|A|$. The expected number of sensor nodes on the segment in Figure 3 (a) is then given by:

$$\frac{\lambda\sqrt{3}R}{2\cos\theta}$$

We further assume that the sensor nodes on the radial segment is uniformly distributed. Then $\frac{1}{\lambda}$ will be the distance between two consecutive nodes on a segment. The total amount of power consumed by all nodes on the segment corresponding to θ, denoted as $E(\theta)$, in one round of transmission can then be calculated as:

$$\begin{aligned} E(\theta) &= \sum_{i=1}^{\frac{\lambda\sqrt{3}R}{2\cos\theta}} \left(\frac{i}{\lambda}\right)^2 \\ &= \frac{R\ (\ 3\sqrt{3}R^2\lambda^2 + 9R\lambda\cos\theta + 2\sqrt{3}\cos^2\theta\ )}{24\cdot\lambda\cdot\cos^3\theta} \end{aligned}$$

The total power consumption for the whole hexagonal cluster follows:

$$\begin{aligned} E_R &= 12\int_0^{\frac{\pi}{6}} E(\theta)\ d\theta \\ &= \frac{\sqrt{3}R\ (4\lambda^2R^2 + 3\lambda^2R^2\ln 3 + 12\lambda R + 4\ln 3)}{8\lambda} \end{aligned}$$

Figure 4 illustrates the trends of E_R as R and λ increases.

3.3 $\frac{R}{2}$ subdivision

The idea of the subdivision, illustrated in Figure 5, is to divide a cluster into a set of smaller hexagons, so that data from some sensor nodes can be transmitted to a closer sub-clusterhead, rather than all to the original clusterhead, resulting in less overall power consumed in transmission.

For instance, as shown in Figure 5, we can divide the original cluster into a group of 1 sub-hexagon and 6 half-sub-hexagons. The data collection of the original clusterhead is now performed in two phases.

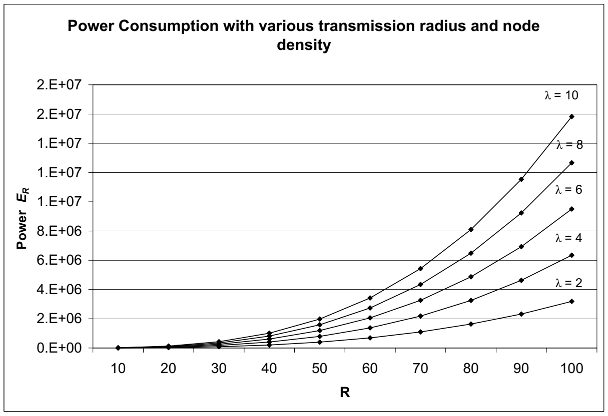

Figure 4: The estimation of power consumption as a function of transmission radius (R) and node density (λ).

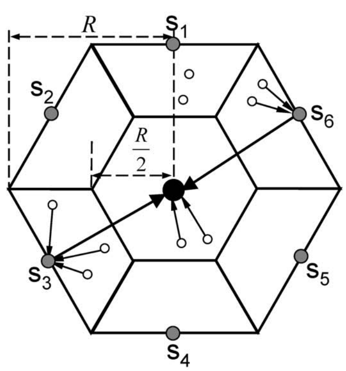

Figure 5: The cluster is divided into 1 sub-hexagon and 6 half-sub-hexagons. The radius of the sub-hexagon is half of the original cluster.

In the first phase, all sub-clusterheads (s_1–s_6 in Figure 5) collect data from sensors in their own sub-clusters. The data is aggregated and/or preliminarily processed by sub-clusterheads. In the second phase, the original clusterhead collects data from all sub-clusterheads. Instead of receiving data from all nodes in the cluster, the clusterhead now receives data only from nodes in its own sub-cluster, and from 6 sub-clusterheads.

We calculate the power saving if we subdivide the cluster as shown in Figure 5. The total power is equal to the sum of powers in 4 sub-hexagons (1 sub-hexagon plus 6 half-sub-hexagons), plus the power used by s_1–s_6 to transmit the aggregated data. We assume for now a "deep" aggregation. That is, s_1–s_6 almost completely absorb the collected data, and as a result send the clusterhead just one unit (e.g., one packet) of data. We will discuss later what if this is not the case.

Substituting $\frac{R}{2}$ into the place of R in the expression for E_R, we can get the total power for 4 sub-hexagons:

$$4 \times \frac{\sqrt{3}(\frac{R}{2})\ [4\lambda^2(\frac{R}{2})^2 + 3\lambda^2(\frac{R}{2})^2 \ln 3 + 12\lambda(\frac{R}{2}) + 4\ln 3]}{8\lambda}$$

$$= \frac{\sqrt{3}R\ (\lambda^2R^2 + \frac{3}{4}\lambda^2R^2 \ln 3 + 6\lambda R + 4\ln 3)}{4\lambda} \quad (2)$$

For the 6 sub-clusterheads s_1–s_6, the total power to transmit to the clusterhead is:

$$6 \times (\frac{\sqrt{3}R}{2})^2 = \frac{9R^2}{2} \quad (3)$$

The sum of (2) and (3) is the approximation of the total consumed power after the subdivision:

$$E_{\frac{R}{2}} = \frac{\sqrt{3}R\ (\lambda^2R^2 + \frac{3}{4}\lambda^2R^2 \ln 3 + 6\lambda R + 4\ln 3)}{4\lambda} + 4.5R^2$$

Plotted in Figure 6, $\frac{E_R - E_{R/2}}{E_R}$ is an indication, in terms of percentage, how much power can be saved after the subdivision. The value approaches 50% as R grows. The larger the node density λ, the faster the saving rate reaches the 50% limit.

3.4 Multilevel subdivisions

It can be shown that a deeper subdivision can further reduce the overall power consumption of the hexagonal cluster. Figure 7 illustrates $\frac{R}{3}$ and $\frac{R}{4}$ subdivisions.

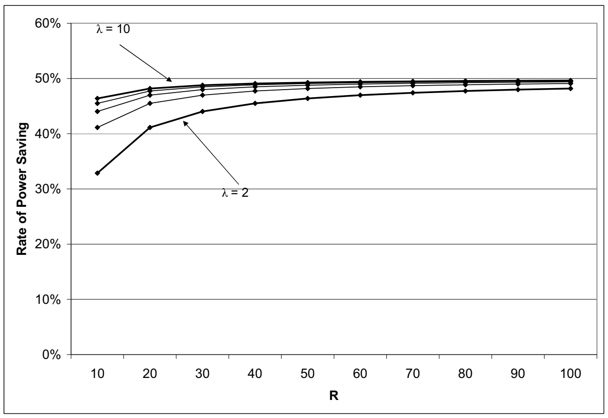

Figure 6: Saving rate $(E_R - E_{\frac{R}{2}})/E_R$ for $\frac{R}{2}$ subdivisions for various densities: the bigger the λ (node density), the faster the power saving rate reaches the 50% limit.

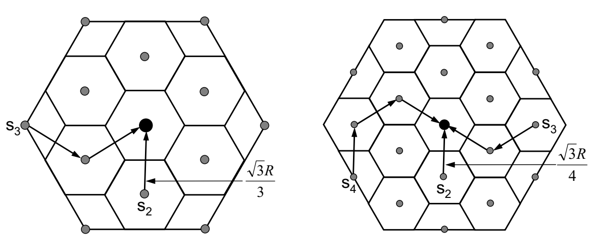

Figure 7: $\frac{R}{3}$ and $\frac{R}{4}$ subdivisions.

However, there exists a limit how deep the division can be carried out while still benefiting from it. In this subsection, we will get an estimate for this maximal, as well as optimal, level-limit.

We assume that sub-clusterheads' packets are relayed to the master clusterhead. (Note that multi-hop relay will cost less power than single-hop, direct transmission, due to the fact that the transmission power is proportional to the square of the distance.) For example, in the $\frac{R}{4}$ subdivision in Figure 7, the power for a direct transmission from s_4 to clusterhead would be $(\frac{\sqrt{3}R}{2})^2 = 0.75R^2$, while the 3-hop transmission needs $3(\frac{\sqrt{3}R}{4})^2 = 0.5625R^2$.

The total consumed power of a $\frac{R}{n}$-subdivision consists of two parts:

- *Intra-cluster power:* The total power for transmission from sensor nodes to sub-clusterheads in all sub-clusters;
- *Inter-cluster power:* The total power for transmission from all sub-clusterheads to the clusterhead.

3.4.1 Intra-cluster power

In Figure 5, the $\frac{R}{2}$-subdivision results in 1 whole sub-cluster and 6 half sub-clusters, totalling $4 = 2^2$ whole sub-clusters; in Figure 7 (a), the $\frac{R}{3}$-subdivision results in 7 whole sub-clusters and 6 one-third sub-clusters, totalling $9 = 3^2$ whole sub-clusters; in Figure 7 (b), 13 whole sub-clusters and 6 half sub-clusters, totalling $16 = 4^2$ whole sub-clusters.

Proposition 1. *There are totally an equivalent of n^2 sub-hexagons in a $\frac{R}{n}$-subdivision.*

Proof. See Figure 8.

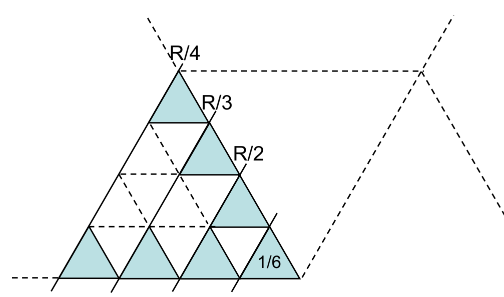

Figure 8: Proof of Proposition 1.

Figure 8 shows the one-sixth of a hexagon for a $\frac{R}{n}$-subdivision, $n = 2, 3, 4, ...$ Therefore each small triangle will represent a sub-hexagon in the $\frac{R}{n}$- subdivision. There are $1+3+5+\cdots+(2n-1) = n^2$ triangles. Thus there are an equivalent of n^2 small hexagons in a $\frac{R}{n}$-subdivision.

■

The total intra-cluster power can then be calculated as follows:

$$n^2 \cdot \frac{\sqrt{3}(\frac{R}{n})\ [4\lambda^2(\frac{R}{n})^2 + 3\lambda^2(\frac{R}{n})^2 \ln 3 + 12\lambda(\frac{R}{n}) + 4\ln 3]}{8\lambda}$$

$$= \frac{\sqrt{3}R\ (4\lambda^2 R^2 + +3\lambda^2 R^2 \ln 3 + 12n\lambda R + 4n^2 \ln 3)}{8n\lambda} \quad (4)$$

3.4.2 Inter-cluster power

To determine the number of sub-clusterheads that relay the data, refer to Figure 9 that illustrates the allotment of sub-clusterheads for various subdivisions in the one-sixth of a hexagon.

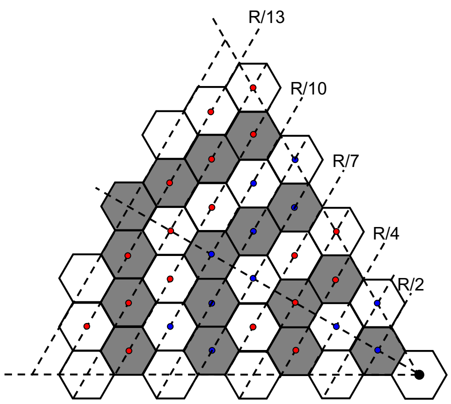

Figure 9: Sub-clusterheads for various subdivisions in the $\frac{1}{6}$ of a hexagon.

It can be proved, and can be observed from Figure 9, that as the levels of subdivision n increase, the relaying sub-clusterheads (in a one-sixth of hexagon) increase by 1 for every 3 levels. The number of sub-clusterheads at level k is then given by:

$$6\lfloor\frac{k+1}{3}\rfloor$$

Assuming that every sub-clusterhead aggregates all collected data and transmits one aggregated packet to the clusterhead, the total transmission power can be expressed by:

$$\sum_{k=2}^{n} 6\lfloor\frac{k+1}{3}\rfloor \cdot (k-1) \cdot (\frac{\sqrt{3}R}{n})^2$$

where $\frac{\sqrt{3}R}{n}$ is the distance of a 1-hop transmission, and $(k-1)$ represents the number of relay hops from a sub-clusterhead at level k to the clusterhead. Since the expression contains a floor function ($\lfloor\frac{k+1}{3}\rfloor$) which poses difficulty in manipulation, we resort to an *upper-bound* of the power cost as follows:

$$\begin{aligned}
&\sum_{k=2}^{n} 6\lfloor\frac{k+1}{3}\rfloor \cdot (k-1) \cdot (\frac{\sqrt{3}R}{n})^2 \\
&= 6(\frac{\sqrt{3}R}{n})^2 \sum_{k=2}^{n} \lfloor\frac{k+1}{3}\rfloor \cdot (k-1) \\
&< 6(\frac{\sqrt{3}R}{n})^2 \sum_{k=2}^{n} (\frac{k+1}{3}) \cdot (k-1) \\
&= \frac{R^2(2n^2+3n-5)}{n} \qquad (5)
\end{aligned}$$

3.4.3 Total transmission power and subdivision limit

The total transmission power after subdivision is just the sum of intra-cluster and inter-cluster power:

$$E(R,\lambda,n) = (4) + (5).$$

Figure 10 plots the value of $E(R,\lambda,n)$ as a function of n, for $R = 50$, and $\lambda = 2, 4$, respectively.

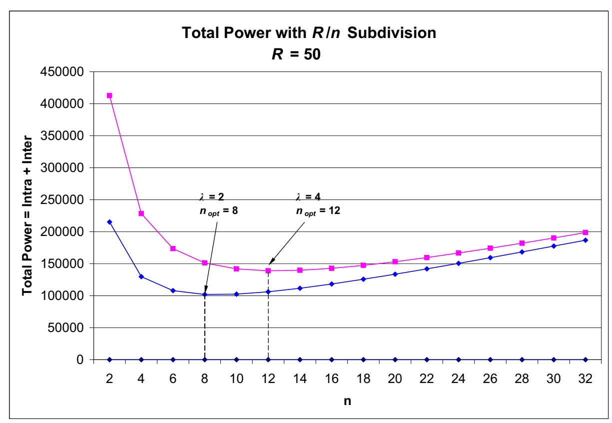

Figure 10: $E(R,\lambda,n)$ as a function of n, for $R = 50$, and $\lambda = 2, 4$.

Figure 10 shows that there exists a subdivision limit n_{opt}, such that for any $n > n_{opt}$, the cluster no longer benefits from the $\frac{R}{n}$-subdivision. Also observed from Figure 10 is that the upper-limit n_{opt} increases as node density λ increases. To get an estimate of n_{opt}, we can take the derivative of $E(R,\lambda,n)$, and solve

$$\frac{\partial E(R,\lambda,n)}{\partial n} = 0$$

for n.

The expression of $\frac{\partial E(R,\lambda,n)}{\partial n}$ is too tedious to be copied here. We just give the solution of $\frac{\partial E(R,\lambda,n)}{\partial n} = 0$ for n, which is the optimal levels of subdivision in terms of R and λ:

$$n_{opt} = \frac{1}{2}\frac{\sqrt{(\sqrt{3}\ln 3 + 4R\lambda)R\lambda(-40 + 4R\sqrt{3}\lambda + 3\sqrt{3}R\lambda \ln 3)}}{\sqrt{3}\ln 3 + 4R\lambda}$$

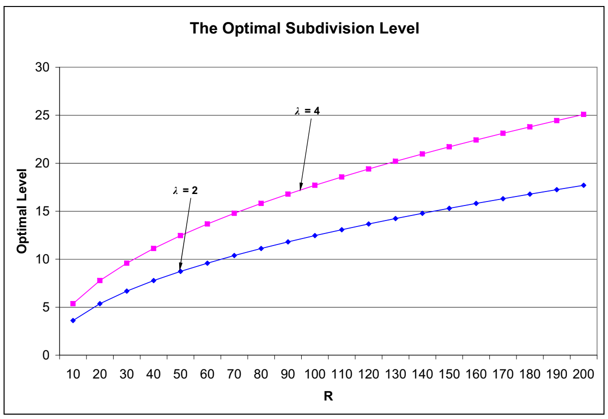

Figure 11: Optimal level number in terms of R and λ, for $\lambda = 2, 4$.

Figure 11 plots the growth of n_{opt} as a function of R and λ. It shows that n_{opt}, the theoretical optimal division levels, increases as R and λ increase. That is, the larger the R and λ, the deeper can the hexagon be subdivided while still benefiting from the subdivision. However, n_{opt} is the maximum level the subdivision can go. Any further subdivision beyond $\frac{R}{n_{opt}}$ will not benefit more in power saving.

3.5 Multi-packets from sub-clusterheads

The preceding analysis assumes that all sub-clusterheads have completely absorbed (preprocessed) the collected data, and send the clusterhead just one packet of data. There are many applications that fit this scenario. We now consider where this is not the case. That is, the data collected by sub-clusterheads cannot be completely aggregated, and has to be sent to the clusterhead in more than one packet. Obviously, this will adversely affect the power saving brought about by subdivision.

To quantitatively analyze the effect of multi-packets on power saving, we introduce a variable, ξ, to represent the *average number* of packets a sub-clusterhead sends to the clusterhead. The value of ξ starts from 1, and is associated with the sub-clusterheads. Multiplying ξ with (5), the expression

$$\frac{R^2(2n^2 + 3n - 5)}{n} \cdot \xi \qquad (5')$$

represents the power used by all sub-clusterheads each of which sending ξ packets to the central head. The total power taking into account the cost of multi-packets can then be expressed by:

$$E(R, \lambda, n) = (4) + (5').$$

The previous analysis assumed $\xi = 1$. When ξ grows, obviously the power saving decreases. It is therefore of interest to know what is a *threshold* ξ_t, such that when $\xi > \xi_t$, the cluster no longer benefits from the subdivision. We look at $\frac{R}{2}$ first. We solve for $\xi_{\frac{R}{2}}$ in the following inequality:

$$E(R, \lambda, 2) = [\ (4) + (5')\ ]_{n=2} \geq E_R$$

which results in:

$$\xi_{\frac{R}{2}} \geq \frac{\sqrt{3}(4\lambda^2R^2 + 3\lambda^2R^2\ln 3 - 8\ln 3)}{72R\lambda} \qquad (6)$$

Expression (6) above shows that $\xi_{\frac{R}{2}}$ is almost linearly proportional to the product of R and λ. We plot in Figure 12 the threshold value $\xi_{\frac{R}{2}}$ for a range of R and λ.

To see the trend of ξ as subdivision deepens, we solve ξ for

$$E(R, \lambda, n) \geq E_R$$

and get:

$$\xi_{\frac{R}{n}} \geq \frac{\sqrt{3}(-4n\ln 3 + 4\lambda^2R^2 + 3\lambda^2R^2\ln 3)}{8\lambda R(2n+5)} \qquad (7)$$

which is a descending function of n. The threshold packet number ξ as a function of subdivision level n (for $\lambda = 10$ and $R = 50$) is plotted in Figure 13.

The interpretation of (6)-(7) (Figures 12-13) is that

1. The threshold of average packet number (from each sub-clusterhead) depends on the node density, as well as the transmission range of the clusterhead. The larger the product of the two, the more packets a sub-clusterhead can send while still benefiting from the subdivision;

   and

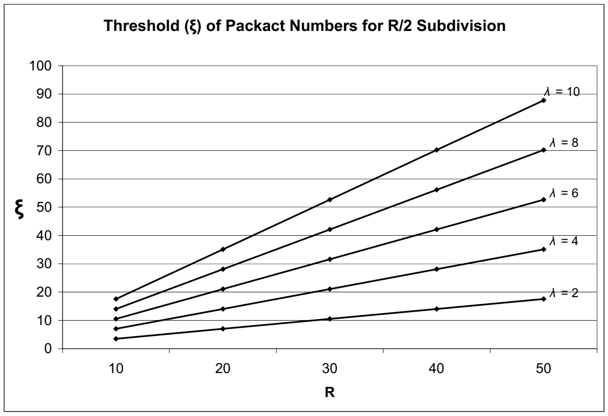

Figure 12: The figure illustrates the packet threshold for $\frac{R}{2}$ subdivision. ξ is the theoretical maximal number of packets a sub-clusterhead can send to the clusterhead while still benefiting from the subdivision. The more the value of R and λ, the more packets a sub-clusterhead can send.

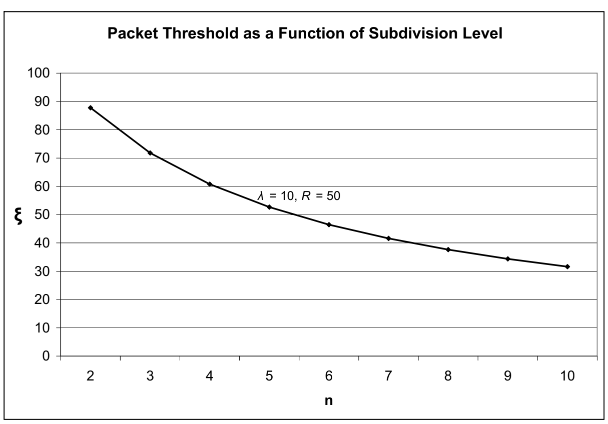

Figure 13: n is the subdivision level. The deeper the subdivision, the fewer packets a sub-clusterhead can send to the clusterhead while still benefiting from the subdivision.

2. As the subdivision deepens, fewer and fewer packets can be sent from sub-clusterheads while still benefiting from the subdivision. In addition to the overhead and reduced gain percentage, this should be another tradeoff factor to take into account when deepening the subdivision.

For example, for the $\frac{R}{2}$ subdivision (Figure 12), node density $\lambda = 10$ with transmission range $R = 30$ would allow up to 52 packets to be sent by each sub-clusterhead; for $R = 50$, each sub-clusterhead can send up to 87 packets. However, for the same transmission range $R = 50$, $\frac{R}{3}$ and $\frac{R}{4}$ subdivisions (see Figures 13) would allow fewer (71 and 60, respectively) packets to be sent. This analytical result sits well with our intuition: deeper subdivision will need more sub-clusterheads, which means more nodes will be involved in transmission of longer distance. As a result, fewer packets can be sent by each sub-clusterhead.

4 Concluding remarks

Power-efficiency is a very crucial issue in the design of wirelessly networked systems. In this paper we have addressed the issue of reducing the cluster's overall transmission power in a hexagonally clustered WSN. We use a group of sub-clusterheads in the cluster to help carry out the basic task of sensor data collection/aggregation, so that most nodes transmit data to an "auxiliary" subhead closer than the original clusterhead. Assuming a spatial Poisson distribution of sensor nodes in the cluster, we discussed the beneficial effect of subdivision, performing an analytical estimate of power saving as the subdivision deepens. We also analyzed the scenario where multiple packets are sent. In spite of the specific assumption in the analysis that the WSN nodes are deployed according to spatial Poisson distribution, the results can shed light on the prospect of using subdivision as a feasible means to prolong the lifetime of sensor nodes.

Since subdivision as a means of power saving has been addressed only analytically in this paper, an immediate follow-up of the theoretical results would be a simulation study to evaluate the relevance and efficiency of the subdivision schemes. The analysis in the current work assumed a hexagonally clustered WSN, in which sensors are assumed to follow spatial Poisson distribution. Similar approach of subdivision can be applied to other cluster shapes, such as square and circle, as well as to other node distributions. A general framework for subdividing clusters for power saving could be worked out through a comprehensive study of subdivision of various clusters and node distributions.

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