Probabilistic-based Message Dissemination in Ad-Hoc and Sensor Networks using Directional Antennas

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***Abstract*—Gossiping is a probabilistic approach to facilitate routing and information dissemination in ad hoc and sensor networks. In this paper, we provide a comprehensive evaluation of gossip-based protocols using directional antennas. We evaluate different aspects of the directional gossiping protocols: directional gossiping mechanism, network graph type, transceiver antenna mode, et cetera. For instance, we analyze the gossiping performance by using different antenna modes: directional-transmitting with omnidirectional-receiving (D-O) and directional-transmitting with directional-receiving (D-D). We show the appropriate antenna beamwidth range for using different antenna modes. Moreover, we also discuss and compare several directional gossiping extension algorithms. Our work provides novel and valuable insights for using directional antennas in ad hoc and sensor networks.**

I. Introduction

Gossiping is a well-known probabilistic mechanism for information dissemination within communication networks. By using gossiping approach, each node in the network forwards packets with a pre-specified probability p, which is called as *gossiping probability*, or has a probability $1-p$ to discard it. Compared to flooding, it is a simple alternative to mitigate the amount of generated overhead, which is termed as *broadcast storm problem* [1]. Hence, gossiping is more efficient and scalable than flooding. Gossiping is widely used to facilitate routing protocols and optimize broadcasting process in ad hoc and sensor network. Extensive work has been done on this topic, e.g. [1]−[6]. In [2], Haas *et al.* presented GOSSIP, which introduces a parameter k to tune the gossiping probability. They define that all the nodes located within k hops from the source node always forward packets with probability 1. GOSSIP further uses two forwarding probabilities $p1$ and $p2$, instead of a single probability p, where $p2 > p1$. A packet is forwarded with $p2$ if the number of its neighbor is below certain threshold. Otherwise, it is forwarded with $p1$. In [3], Kyasanur *et al.* proposed a gossip-based protocol which automatically adapts gossiping probability according to the network topology. Cartigny and Simplot [6] combined the forwarding probability with distance information to enable network density aware gossiping. All the work mentioned above is based on omni-directional antennas.

By concentrating energy on a certain direction, directional antennas could efficiently use power for transmission, and meanwhile avoid interference from other directions. Therefore, directional antennas increase the robustness of a network. However, using directional antennas for broadcasting is not a trivial issue, because a node should sweep a packet in each possible direction, which makes the broadcast storm problem even worse. To reduce directional broadcasting overhead, the authors in [7] proposed a relay node selection mechanism, which defines that only the farthest neighbor in a certain direction forwards broadcasting packets. However, their work assumed a preliminary neighbor discovery process, so each node knows if it is the farthest neighbor of the node from which it receives the packets. In [8], the authors investigated directional routing protocols in ad hoc networks. To reduce routing overhead, they proposed to avoid forwarding route request in the direction where the channel is busy. In [9], Shen *et al.* discussed a simple probabilistic-based approach for message forwarding with directional antennas. Compared to the related work, our work has the following contributions. First, we investigate multiple directional gossiping mechanisms based on various network graphs (random graph, lattice graph, and random geometric graph). Second, we provide the gossiping performance by using different combinations of transceiver antenna mode and variant antenna beamwidth. We show the appropriate antenna beamwidth range for using different antenna modes. Moreover, we discuss several gossiping extension schemes based on directional antennas. We demonstrate that a combination of two gossiping optimization mechanisms could further enhance the gossiping performance. To the best of our knowledge, this is the first time that directional antennas are comprehensively evaluated in gossip-based protocols. Our work provides valuable insights for using directional antennas in ad hoc and sensor networks.

The rest of this paper is organized as follows: in Section II, we provide the related rationale and assumptions used in this work. In Section III, we describe and analyze the performance of directional gossiping algorithms from different aspects. Several gossip-based extension protocols are compared in Section IV. Finally, we conclude the paper in Section V.

978-1-4244-5113-5/09/$25.00

II. Preliminaries

A. Percolation Theory and Phase Transition Phenomenon

Percolation theory is used to model the behavior of a random medium. While considering the medium as a number of nodes in a network, percolation probability $\theta(p)$ is defined as the probability that a given node belongs to an infinite cluster. There exists a critical threshold p_c for gossiping probability p [10]. If the gossiping probability is larger than p_c, it is almost sure to guarantee the dissemination of information in the entire network, such that

$$\theta(p) \begin{cases} = 0 & \text{if } p < p_c, \\ > 0 & \text{if } p > p_c. \end{cases} \tag{1}$$

This phenomenon is termed as *bimodal* or *phase transition* behavior in percolation theory [4][10]. This phenomenon is also observed from gossip-based protocols [2]. In [9], the authors declared that if the network is sufficiently large, the gossiping probability p of omni-directional and directional broadcasts over a network are the same as the percolation threshold of *site percolation*. The explicit expression of the percolation probability is hard to get. It varies according to the network topology, but approximations can be obtained via simulations. For instance in [2], the author illustrated the transition threshold of gossiping probability for omni-directional broadcasting. In this work, we will validate the bimodal behavior by using different transmission and reception antenna modes.

B. Graph Type

Three types of graphs are used in this work, which are shown in Figure 1.

- Random graph (T1)

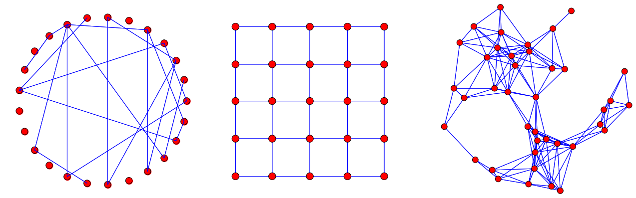
A random graph $G_{p_r}(N)$, consists of N nodes, in which each link between two nodes is chosen independently and with probability p_r. The existence of a link in a random graph does not depend on the position of the transmission pairs. Therefore, a node's position within the network does not influence the network connectivity. In order to demonstrate the directional gossiping protocols on a random graph, we generate N nodes which are uniformly distributed in a circular network with radius R. Each node selects its neighbors from the $N-1$ nodes with a probability p_r. The mean node degree in random graph is calculated as $E[d] = (N-1)\cdot p_r$.

- Lattice graph (T2)

Lattice graph is also known as square grid graph. A 5×5 lattice graph is shown in Figure 1 (b), in which nodes are deployed as square grid.

- Random geometric graph (T3)

Random geometric graph $G_{p_{ij}}(N)$ could be considered as a variation of the random graph, in which a link exists according to a variable probability p_{ij} instead of a fixed probability p_r. Random geometric graph is a good model for ad-hoc wireless networks, because the probability of the existence of a link can be specified by the radio propagation model. In this work,



(a) Random graph **(b) Lattice graph** **(c) Random geometric graph**

Fig. 1. Graph illustrations

for simplicity, we define that a node i has probability 1 to connect to the neighbors within its transmission range d.

$$p_{ij} \begin{cases} = 1 & \text{if } \|i-j\| \le d, \\ = 0 & \text{if } \|i-j\| > d. \end{cases} \tag{2}$$

where, $\|i-j\|$ represents the Euclidean distance between node i and j. To construct a random geometric graph, N nodes are uniformly distributed over a circular area with radius R. Each node links with all the other nodes within its transmission range.

C. Antenna Mode

For adaptive array based directional antenna systems, two operating modes can be used for transmitting and receiving: omni-directional mode and directional mode. By using different transceiver antenna modes, there exist several neighborhood relationships: O-O neighbor, D-O (or O-D) neighbor and D-D neighbor, which are explained in Table I. According to the propagation model in (3), it indicates that a higher antenna gain results in a longer transmission range,

$$P_r \propto \frac{P_t G_t G_r}{d^n}, \tag{3}$$

where P_t and P_r are the transmitted and received power, respectively. d is the distance between the transmitter and the receiver, G_t and G_r are the transmitter and receiver antenna gains, respectively. Factor n is the path loss exponent. We define η as the range extension factor, where $\eta = d_\theta / d_{omni}$, and d_θ is the transmission range while using a directional antenna with beamwidth θ and d_{omni} is the transmission range by using an omni-directional antenna. Considering an idealized flat-top pattern directional antenna, the antenna gain is given as $G = \frac{2\pi}{\theta}$. Therefore, we have the range extension factor $\eta = (G_t G_r)^{\frac{1}{n}}$. For node i, if we denote $\Phi_{OO}(i)$ as the O-O neighbor set of node i, we have $\Phi_{OO}(i) \subset \Phi_{DO}(i) \subset \Phi_{DD}(i)$.

TABLE I
TRANSCEIVER MODE SPECIFIED NEIGHBOR RELATIONSHIPS

	Transmitter antenna	Receiver antenna
O-O neighbor	Omni-directional	Omni-directional
D-O or O-D neighbor	Directional	Omni-directional
	Omni-directional	Directional
D-D neighbor	Directional	Directional

D. Performance Metrics

In this work, we examine the performance of various gossiping schemes based on the following metrics.

1) Giant component size: Giant component size (GCS) indicates the connectivity of a network. Giant component refers to the largest connected cluster in a network. GCS is the fraction of nodes in the network that, in single-hop or multi-hop fashion, are connected to each other. Obviously, when GCS is equal to 1, the entire network is connected.

2) Overhead: Overhead refers to the total number of generated packets in one run of simulation[1]. The amount of overhead indicates the efficiency of a gossiping protocol.

3) Path length: During the gossiping process, messages are transmitted from the source and disseminated to the rest of the network via different routes. The path length refers to the hop count of the longest path in the network, which could represent the latency of the gossiping process.

4) Delivery ratio: We denote $\Pr[A_N(L)]$ as the delivery ratio, which represents the probability that all the nodes within the network are informed when the path length is L. $\Pr[A_N(L)]$ could be considered as the percolation probability in a network with finite size.

E. Assumptions

1) To achieve a homogeneous gossiping environment, the source is located at the center of the network when we use random geometric graph and lattice graph. For random graph and random geometric graph, all the nodes are uniformly distributed within the network.
2) During the gossiping process, a node could receive multiple messages from different routes. A node only reacts to the first received packet, and simply discards all the packets after the first one.
3) All the constructed networks are completely connected. Moreover, a perfect medium access is assumed to eliminate the MAC layer influence. We consider that the network is at the initialization phase, so a node has no information about its surrounding neighbors.
4) All the distances used in this work are normalized value. By normalizing, we mean the ratio between the real distance and the transmission range by using omni-directional antennas. Therefore, the normalized transmission range by using omni-directional antennas is 1.
5) All the simulation results are averaged based on 1000 iterations.

III. Gossiping Using Directional Antennas

The original gossiping protocol defines that after a node receives a packet, it has a probability p to forward the packet, or has a probability $1-p$ to discard the packet. By using directional antennas, the azimuth plane of a node is divided into N_B beam sectors, where $N_B = 2\pi/\theta$. According to different forwarding manners, we define two basic Directional Gossiping (DG) mechanisms: (a) per-sector based DG and (b) per-node based DG.

- Per-sector based DG (DG1): After a node receives a packet, it generates a probability p_i for each beam sector, where $i \in [1, N_B]$. A packet is forwarded from beam sector i, if $p_i < p$.
- Per-node based DG (DG2): After a node receives a packet, it only generates one probability p_v. If $p_v < p$, it forwards a packet in each beam sector. Otherwise, it does not forward it at all.

In order to guarantee that there are packets sent out from the source, the source has probability 1 to transmit from each beam sector.

[1]We define one run of simulation as the period from the moment that the first packet is transmitted from the source till the gossiping process dies out or the entire network is informed.

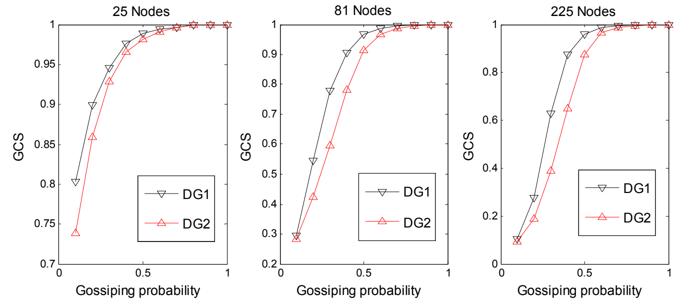


Fig. 2. Performance comparison between DG1 and DG2 in variant size of networks, $\theta = 90°$, D-O mode.

A. DG Mechanism Comparison

We examine the performance of DG1 and DG2 in a random geometric graph. Nodes use directional antennas for transmitting and omni-directional antenna for receiving or listening. We keep the mean network density of around 10.5, and scale up the total number of nodes from 25 to 225. The simulation results are shown in Figure 2. In general, DG1 exhibits a higher GCS than DG2, which means DG1 can inform more nodes in one execution of gossiping process. Therefore, DG1 is more reliable than DG2. In the following simulations, only DG1 is used to examine the directional gossiping performance.

B. Network Graph Influence

In this section, we evaluate the directional gossiping performance in several network graphs. To compare different network graphs, we use node degree as the guide line. Node degree refers to the number of direct neighbors of that node in the network. Therefore, each type of network is formed with the same or similar mean node degree for performance comparison. In a large scale network, the mean node degree can be simply approximated as $(N\left(\frac{r_\theta}{R}\right)^2 - 1)$, where r_θ is the normalized transmission range by using directional antenna with beamwidth θ, N is the total number of nodes in the network, and R is the normalized radius of the circular network. When the network radius R is not large enough, the border effect should be taken into consideration and the mean node degree is approximated as:

$$E[d] \approx \left(\int_0^{R-r_\theta} f(x)dx + \int_{R-r_\theta}^{R} \phi(r_\theta + x - R) f(x)dx \right) \times \left[N \left(\frac{r_\theta}{R} \right)^2 - 1 \right], \tag{4}$$

where $f(x)$ is the position probability density functions of the node, and $f(x) = 2x/R^2$. Function $\phi(.)$ represents the proportion of the effective coverage area of a node at the network border area, and we have

$$\phi(d) \approx (1 - \frac{\varphi}{\pi}) + \frac{1}{\pi}(1 - \frac{d}{r_\theta}) \sin(\varphi). \tag{5}$$

The deduction of (5) is given in Appendix A. In a lattice graph, the distances between adjacent nodes are 1. When using $90°$ directional antennas, the mean node degree is 10.68 for lattice graph, which is measured from the simulation. For the random geometric graph, the network radius is set to 8.36 to get similar mean node degree as the lattice graph by using $90°$ directional antennas, in which the mean node degree is 10.69 according to (4). For random graph, the link selection probability p_r is set to 0.0477. According to the above setting, we illustrate the influence from the network graph on the directional gossiping performance as shown in Figure 3. In Figure 3 (a), we compare the GCS by using DG1 in different network graphs with $90°$ directional antenna. In Figure 3 (b), we set the gossiping probability to 1 and vary the antenna beamwidth to get path length performance. The DG1 algorithm in random graph exhibits the highest GCS and the shortest path length. That is because, in random graph, the connection between any two nodes depends on the link selection probability p_r. Therefore, the possibility for a node to be reached by the source is independent of the position of the node. In the random geometric graph, the distance between a node and the source influences the latency that the node is informed. For instance, if a node is far away from the source, it has to wait until the packet reaches the node via multiple relaying nodes. Therefore, the message dissemination speed is faster in random graph than in random geometric graph. The performance of DG1 in lattice graph is in between the other two graphs.

C. Transceiver Antenna Mode Influence

Directional broadcasting is not as efficient as omni-directional broadcasting, since a node duplicates a broadcasting packet and transmits one in each beam sector in directional broadcasting. However, directional antennas can achieve longer transmission range, so more neighbors could be covered by one round of sweeping. We demonstrate the performance by using different transceiver antenna mode (D-O and D-D) in Figure 4. D-O and D-D induce similar amount of overhead in the gossiping progress, but using D-D mode achieves higher GCS compared to D-O mode, especially when the gossiping probability is low. That is because although D-D mode limits the transmitting and receiving directions, but the

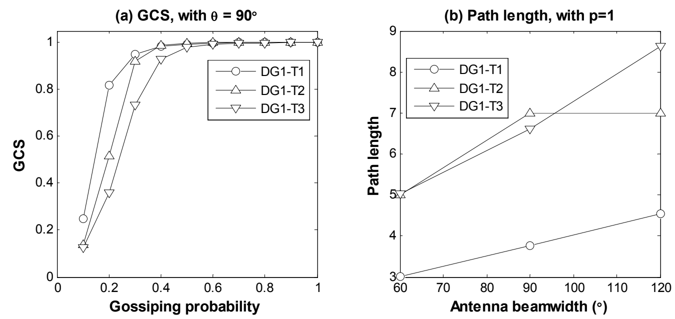


Fig. 3. Network graph influence on directional gossiping performance, based on D-O mode, $N = 225$, $R = 8.36$.

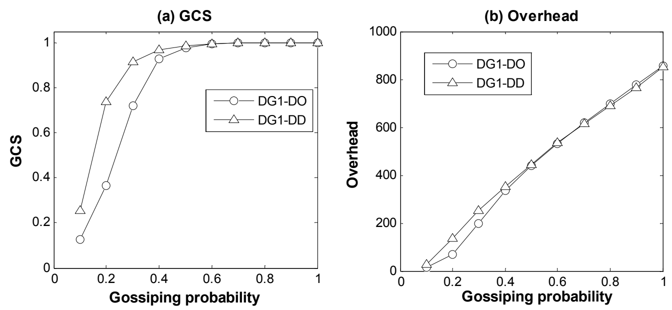


Fig. 4. Transceiver mode impact on the directional gossiping performance in random geometric graph, $N = 225$, $R = 8.36$, and $\theta = 90°$.

gossiping performance can be compensated by the increase in the transmission range.

Figure 5 (a) depicts the relationship between the gossiping probability, the resultant path length and the delivery ratio by using DG1. From this figure we can observe that, to achieve the same delivery ratio Pr$[A_N(L)]$, a lower gossiping probability leads to a longer path length. When the gossiping probability is lower than 0.3, the network cannot be totally covered at all, which verifies the bimodal behavior of the directional gossiping process. In contrast to Figure 5 (a), we plot the resultant path length and the delivery ratio by using D-D mode with varying gossiping probabilities. The performance is shown in Figure 5 (b). From this figure we can observe that, compared to D-O mode, using D-D mode for directional gossiping results shorter path length. Similar to D-O mode, when the gossiping probability is lower than 0.4, the network cannot be totally informed at all, which also validates the bimodal behavior of the directional gossiping performance in D-D mode. Figure 4 indicates that, using D-D mode could achieve higher GCS than using D-O mode, especially when the gossiping probability is low. However, an interesting observation from Figure 5 is that, the probability Pr$[A_N(L)]$ measured by using D-D mode is lower than using D-O mode. For instance, when the gossiping probability $p = 1$, the probability to fully inform the network is only 92.4% by using D-D mode. In contrast, D-O mode can fully inform the network with probability 100%. Therefore, we can conclude that, compared to D-O mode, using D-D mode has faster message dissemination speed and fair message coverage capability,

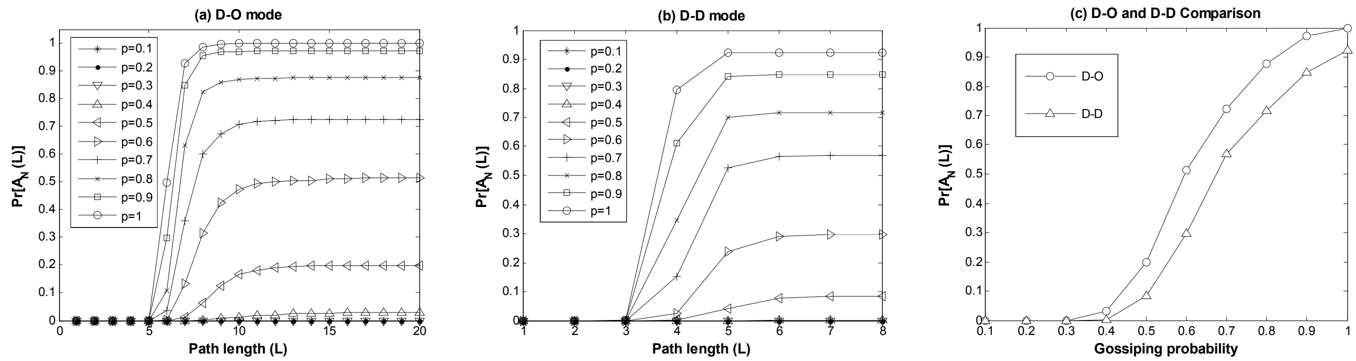


Fig. 5. Influence of the gossiping probability on the probability that all the nodes within the network are informed, with $N = 225$, $R = 8.36$, and $\theta = 90°$.

but it is difficult for D-D mode to completely cover the entire network, which means, when the gossiping probability is high, the resultant GCS by using D-D mode could be quite close to 100% but difficult to reach 100%.

To better illustrate the phase transition behavior, we plot the $\Pr[A_N(L)]$ of the longest path length by using D-O mode and D-D mode in Figure 5 (c), which confirms the critical threshold p_c in the percolation theory mentioned in (1).

D. Antenna Beamwidth Influence

We have already explained the relation between antenna beamwidth and transmission range in previous section. In Figure 6 we show the antenna beamwidth influence on DG1 algorithm in random geometric graph by using D-O mode and D-D mode. The gossiping probability is set to a fixed value 0.5. These two transceiver antenna modes behave in a piecewise manner. When antenna beamwidth is narrow, like $\theta < 20°$, D-O mode exhibits higher GCS than D-D mode, and also higher overhead. It should be noticed that, the lower amount of overhead by using D-D mode is because of the dying out of the gossiping process other than the efficient transmission. When antenna beamwidth is within the range $20° \leq \theta \leq 60°$, using D-O and D-D modes achieve similar connectivity and overhead. When antenna beamwidth is bigger than $60°$, using D-D mode obtains better connectivity and both of these transceiver antenna modes have similar amount of overhead. Therefore, using D-D mode is not suitable for small antenna beamwidth. When antenna beamwidth is within a moderate range, using D-D mode is better than D-O mode, because they can use the similar amount of overhead to achieve the same connectivity, but D-D mode results shorter path length.

IV. Directional Gossiping Extensions

To increase the efficiency of a gossiping process, some gossiping extension ideas are proposed in the literature. In this section, we will explain two gossiping extension mechanisms based on DG1: distance and angle based optimization and k-hop based gossiping algorithm.

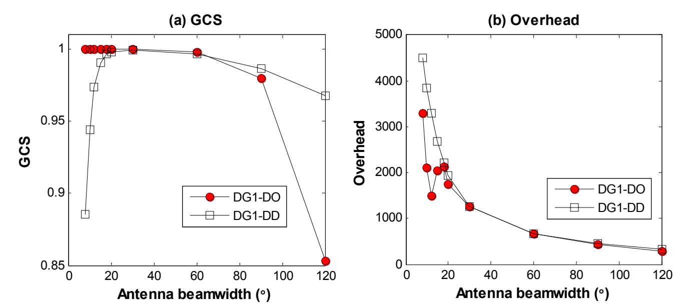


Fig. 6. Performance comparison between D-O mode and D-D mode with variant antenna beamwidth, gossiping probability p is 0.5, $N = 225$, $R = 8.36$.

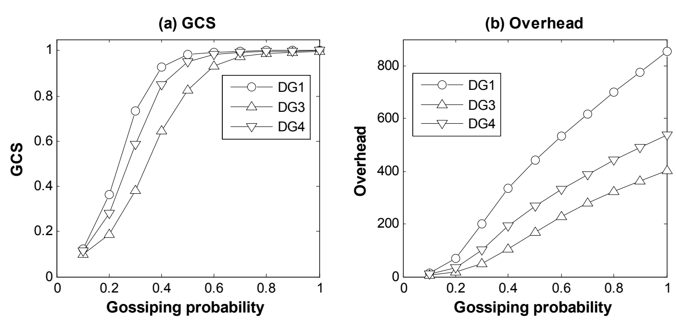


Fig. 7. Performance comparison between DG3 and DG4, D-O mode, $N = 225$, $R = 8.36$, $\theta = 90°$.

A. Distance and Angle based Optimization (DG3)

After receiving a packet, the distance between transmitter and receiver can be estimated according to the received signal strength. In [1], the authors proposed a distance-based broadcasting optimization scheme. They proposed that, if the estimated distance is smaller than a certain distance threshold, the received packet will not be broadcasted from the receiver. This idea is extended in [9], in which authors declared that a node that covers a bigger range has a higher probability of forwarding. They proposed a forwarding probability calculation method, which depends on the source-destination distance and the packet forwarding direction. In our work, we combine this distance and angle based optimization schemes with gossiping protocol. Therefore, each node could have different gossiping probability which depends on the distance and relative direction between a node and its previous transmitter.

When a node receives a packet from beam sector n_r, based on the received signal strength, the node estimates the distance D between the transmitter and itself. This node computes a gossiping probability for each beam sector, which is a product between the basic gossiping probability and a weighting factor. The weighting factor w_i for beam sector n_i is expressed as:

$$w_i = 1 - \left(\sqrt{1 - \left(\frac{D \sin(\Delta n_i \theta)}{r_\theta} \right)^2} - \frac{D \cos(\Delta n_i \theta)}{r_\theta} \right)^2, \tag{6}$$

where $\Delta n_i = \min [|n_i - n_r|, N_B - |n_i - n_r|]$, Δn_i is the beam sector difference between the receiving sector n_r and the forwarding sector n_i. r_θ is the normalized transmission range. Weighting factor w_i should be a value that is bigger than 0. If the calculated value of w_i is smaller than 0, w_i is simply reset to 0. The forwarding probability is calculated as $p_f = p \cdot w_i$.

B. Simplified Distance and Angle based Optimization (DG4)

DG4 is a simplified version of DG3. Based on the estimated distance, a node estimates a threshold angle θ_{TH}, where $\theta_{TH} = \arccos\left(\frac{D}{2r_\theta}\right)$, if the packet forwarding direction θ_i is bigger than θ_{TH}, the weighting factor w_i is 1, otherwise, w_i is set to 0,

$$w_i = \begin{cases} 1 & \theta_i > \theta_{TH} \\ 0 & \text{otherwise} \end{cases}, \tag{7}$$

where $\theta_i = \Delta n_i \theta - \frac{1}{2}\theta$. The forwarding probability is calculated as $p_f = p \cdot w_i$.

Figure 7 (a) and (b) show the comparison results of DG1, DG3 and DG4 in random geometric graph by using D-O mode. It is observed that, DG4 achieves slightly lower GCS than DG1, but it significantly reduces the amount of overhead. For instance, when the gossiping probability is 0.7, the resultant GCSs by using DG1 and DG4 are 99.72% and 99.36%, respectively. But the resultant overhead by using DG4 is only 62.78% of the amount of overhead generated by using DG1. Compared to DG1 and DG4, DG3 has the worst performance in terms of GCS. However, when the gossiping probability is high, e.g. $p > 0.8$, the resultant GCSs are all close to 1, but DG3 has the lowest overhead.

C. DG4 Combined with 'k-hop'

The source node is located in the center of the network, and the other nodes are uniformly distributed within the network. If the source has a smaller number of neighbors, the gossiping process could probably die very fast. To avoid this phenomenon, the authors in [2] proposed a k-hop gossiping protocol. They set the gossiping probability to be 1 for the first k hops from the source, and then the gossiping process continues with probability p since the $k+1$ hop. We combine this mechanism with DG1 and denoted it as DG1(k, θ), and

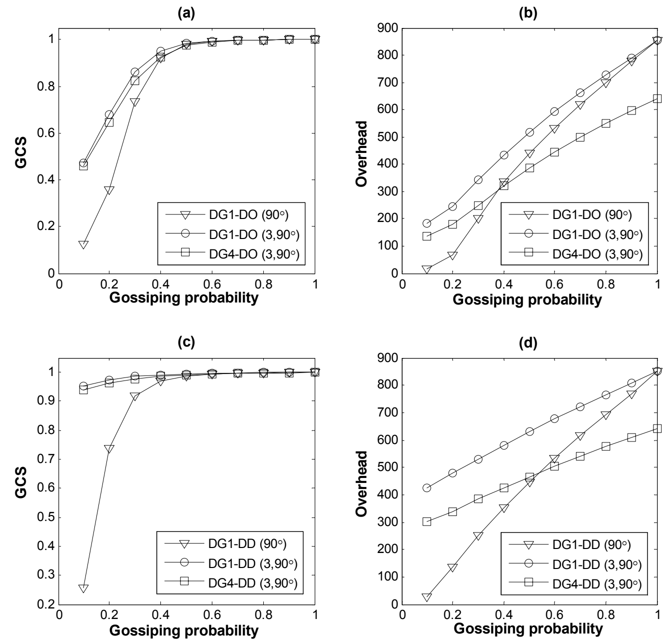


Fig. 8. Performance comparison between k-hop based scheme and the original gossiping protocol, $N = 225$, $R = 8.36$, and $\theta = 90°$.

combine it with DG4 and denoted it as DG4 (k, θ). We compare these two extended schemes with the original DG1, and their performance is compared in Figure 8. Figure 8 (a) and (b) show the results from the schemes using D-O mode, and Figure 8 (c) and (d) show the results from the schemes using D-D mode. It is observed that, compared to DG1, the k-hop scheme enhances the message delivery ratio but also increases the amount of overhead. As we mentioned before, the optimization scheme DG4 could effectively reduce the overhead, but it also reduces the message delivery ratio. When we combine the k-hop scheme and DG4 together as DG4 (k, θ), the new scheme could achieve the similar delivery ratio as DG1(k, θ), but the resultant overhead is much lower. Therefore, the scheme DG4 (k, θ) could inherit the benefit of k-hop scheme and DG4. Figure 8 (a) and (c) also indicate that, using D-D mode with the optimization scheme has better performance than using D-O mode.

V. Conclusion

In this paper, we presented a comprehensive investigation on the gossiping protocols using directional antennas. First, we compared two directional gossiping (DG) mechanisms: per-sector based DG and per-node based DG. We found out that per-sector based DG could achieve higher network connectivity than the per-node based DG. Second, we evaluated the per-sector based DG mechanism in various network graphs, and it is observed that, messages are disseminated fastest in random graph. Third, we investigated the gossiping protocols according to different transceiver modes. We proved that, directional gossiping protocols exhibit the phase transition behavior for both directional transmitting with omni-directional

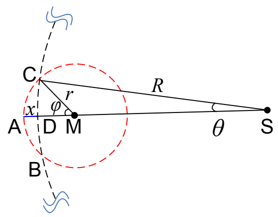


Fig. 9. Border effect illustration

receiving (D-O) mode and directional transmitting with directional receiving (D-D) mode. We also demonstrated the antenna beamwidth influence on the message dissemination performance by using D-O mode and D-D mode. Moreover, we compared the performance of two gossiping extension mechanisms - distance and angle based optimization, and k-hop based optimization. We found that the combination of these two optimization schemes can inherit the benefits of both sides.

APPENDIX A

As shown in Figure 9, a node is located at M and the coverage range of this node is denoted as circle M with radius r. This node is at the border area of a circular network S with radius R. Circle M and S intersect at point B and C. Line MS intersects with Circle M and S at point A and D, respectively. We denote $\|AD\| = x$, $\|DM\| = r - x$, and $\|MS\| = R - r + x$, where $x \in (0, r]$ and the norm is the Euclidean norm. We denote $\|CD\| = d$ and $\angle AMC = \varphi$, where,

$$\begin{array}{l} \varphi = \arccos\left(\frac{r^2+(r-x)^2-d^2}{2r(r-x)}\right) \\ d = \left(2R^2 - 2R^2\left(1 - \frac{2rx-x^2}{2R(R-r+x)}\right)\right)^{\frac{1}{2}} \end{array} \tag{8}$$

The intersection area of Circle M and S is calculated as

$$S_{SM}(x) \approx r^2(\pi - \varphi) + (r - x)r\sin(\varphi) \tag{9}$$

Therefore, the proportion of the effective coverage range of the node is given as $\phi(x) = S_{SM}(x)/(\pi r^2)$.

ACKNOWLEDGMENT

This work was supported by IOP GenCom SiGi Spot project in The Netherlands. Moreover, we would like to thank Siyu Tang and R. Venkatesha Prasad, who provided valuable comments for this work.

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