Reliable Multicast in Wireless Networks Using Network Coding

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***Abstract*—Reliable multicast in wireless networks has been well studied in the sense to solve the feedback implosion issue, which, however, can not reduce the number of retransmissions in order to recover all lost packets at receivers. Most recently, it has been proposed to use network coding for reliable multicast in wireless LANs to reduce the number of retransmissions. In this paper, we propose two new models to further reduce the number of retransmissions for reliable multicast. In the first model, each retransmission encoding decision is made according to the latest "wanted" packet set at all receivers. Thus, the maximum number of receivers can potentially decode out one "wanted" packet from each encoded retransmission packet. Such a model is referred to as *Dynamic Multicast Retransmission Encoding* (DMRE) model. This model is a memoryless model where a receiver will not buffer encoded retransmission packets for later use. In the second model, a receiver will buffer all received encoded retransmission packets and decode out their "wanted" packets at the end of the retransmission batch. Such a model is referred to as *Cache-based Multicast Retransmission Encoding*(CMRE) model. The problem to minimize the number of retransmissions under both DMRE and CMRE models are NP-hard. Effective heuristic algorithms are proposed in this paper. We analyze the impact of packet delivery ratio on the gain of network coding. We derive the lower bound of the expected number of retransmissions using network coding, which provides the insights of the maximum potential gain using network coding in reliable multicast.**

***Index Terms*—Network coding, Reliable multicast, Retransmission**

I. Introduction

We have been enjoying many advantages brought by wireless communication, e.g., mobility and flexibility. However, various factors, e.g., fading, interference, multi-path effects, and collisions, may lead to large loss rate on wireless links, which increases the retransmission overhead. Such retransmission overhead is more severe for multicast in wireless networks.

The retransmission overhead for multicast consists of two parts: (1) the receivers send ACK/NAK feedbacks to the sender; (2) the sender multicasts each lost packet to the multicast group. The ACK/NAK feedbacks may cause a feedback implosion problem. NAK suppression [1], [2] and NAK aggregation [3] are effective approaches to sustain the feedback implosion problem in reliable multicast. Thus, the ACK/NAK overhead can be reduced. Recently, network coding has been proposed as an approach to reduce the retransmission overhead.

Network coding is originally proposed in information theory [4] and has become the most promising approach to improve system throughput in wireless networks. The work in [5] proposed batch retransmission concept for reliable multicast and investigated the gain of using network coding to reduce the number of retransmission packets, thus, to improve the efficiency of retransmission.

Consider a multicast scenario with a sender s and six receivers $r_1, r_2, \ldots, r_6$ as shown in Fig. 1. Suppose that s has sent six packets $p_1, p_2, \ldots, p_6$ to the receivers in the multicast session. Due to wireless link loss and/or congestion loss, the receivers may not receive some packets. As shown in Fig. 1, according to the ACKs/NAKs, the sender s knows that r_1 has lost packet $\{p_1\}$, r_2 has lost packets $\{p_2, p_3, p_4\}$, r_3 has lost packets $\{p_4, p_5, p_6\}$, r_4 has lost packets $\{p_2, p_3, p_5\}$, r_5 has lost packets $\{p_2, p_4, p_6\}$, and r_6 has lost packets $\{p_3, p_5, p_6\}$. Based on such feedbacks, s needs 6 retransmissions in the traditional way, i.e. transmitting $p_1, \ldots, p_6$ at each time slot respectively. However, by the encoding strategy introduced in [5], s needs to transmit 5 packets, $p_1 \oplus p_2$, p_3, p_4, p_5 and p_6 respectively where r_1 can recover p_1 by $p_2 \oplus (p_1 \oplus p_2)$, r_2, r_4 and r_5 can recover p_2 by $p_1 \oplus (p_1 \oplus p_2)$, and etc.

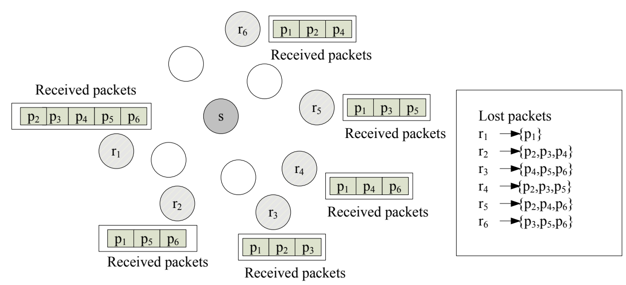


Fig. 1. An example of multicast in wireless networks

When a new batch of retransmission starts, given the information of received packets at each receiver and the set of "wanted" packets at each receiver, the encoding strategy in [5] determines the minimum number of encoded retransmission packets using network coding with the following assumptions:

- The encoding decision for *each* retransmission packet *only* considers the information available at the beginning of each retransmission batch.
- If a receiver receives an encoded retransmission packet and it can not decode any packet in its "wanted" list, it

will throw the encoded retransmission packet away, so called "memoryless" model.

Although such assumptions can make the encoding and decoding easier, the sender can not fully utilize the advantages of network coding. We have the following observations:

- After the first encoded retransmission packet is determined and sent out, some lost packets will be recovered. The receivers will update such information with the sender. When the sender determines how to encode the second retransmission packet, it can make the encoding decision according to the latest "wanted" packet set at receivers. Thus less retransmission packets are required in order to recover all lost packets at the receivers. Under such a strategy, for the example given in Fig. 1, all lost packets can be recovered at all receivers by retransmitting the following 4 encoded retransmission packets $p_4 \oplus p_5$, $p_2 \oplus p_4$, $p_3 \oplus p_5$ and $p_1 \oplus p_6$ in sequence. For example, with $p_4 \oplus p_5$, r_2 and r_5 can recover p_4, r_4 and r_6 can recover p_5. With $p_2 \oplus p_4$, r_2 can recover p_2 according to $p_4 \oplus (p_2 \oplus p_4)$ using p_4 which is recovered before.
- By removing the second assumption, if each receiver can buffer its received encoded retransmission packets, it can accumulate the useful information to recover all its lost packets, which can reduce the number of retransmission packets. Under such a strategy, for the example given in Fig. 1, we only need to retransmit the following 3 encoded retransmission packets, namely, $p_1 \oplus p_2 \oplus p_6$, $p_3 \oplus p_5 \oplus p_6$ and $p_4 \oplus p_5$. When r_5 receives the first encoded retransmission packet, the packet is useless for r_5 because r_5 lost both p_2 and p_6. If r_5 can buffer such received packet, when it receives the second encoded retransmission packet, it can recover p_6 since it already has p_3 and p_5. After p_6 is recovered at r_5, r_5 can use the first encoded retransmission packet to recover p_2 since it has p_6 and p_1 by now.

Such observations motivate this paper which aims to determine the encoding strategy at the sender such that the minimum number of retransmissions is required in order for all receivers to recover their lost packets. Our work mainly focuses on the encoding strategy based on the updated "wanted" packet set at each receiver and we also investigate the encoding strategy if each receiver can buffer its received encoded retransmission packets and recover lost packets when enough encoded retransmission packets are received.

The rest of the paper is organized as follows. Section II introduces the related work. The proposed framework and problem description are given in Section III. In Section IV, we present the algorithm for DMRE model. In Section V, we present the algorithm for CMRE model. In Section VI we analyze the lower bound of the expected number of retransmissions with network coding. Simulation results are given in Section VII. Section VIII concludes the paper.

II. Related work

The wireless broadcasting nature in wireless networks makes it feasible to improve network throughput and achieve energy efficiency with network coding. The network throughput gain using network coding has been studied in [6], [7]. The maximum throughput that a multicast session can achieve with or without network coding in unreliable wireless networks has been studied in [8]. A subgradient-based distributed algorithm using network coding is proposed in [9] to achieve optimal multicast throughput. Energy-efficient broadcast in wireless ad hoc networks using network coding has been studied in [10]. Multicast with the minimum energy consumption in wireless ad hoc networks has been studied in [11] using a linear programming with network coding.

Recently, there has been some work on characterizing the reliability benefit of network coding in lossy networks. It has been shown in [12] that random distributed network coding is asymptotically optimal for wireless networks with and without packet erasures. Lun et al. [13] proposed a capacity-approaching coding scheme for unicast and multicast over lossy packet networks in which all nodes perform random linear coding. M. Ghaderi et al. [14] gave tight asymptotic bounds on the performance of reliability mechanisms based on both ARQ and network coding.

Most of the work above uses random linear coding. Since computation power at each node in a wireless network is limited, we focus the network coding on xor operation, which makes encoding at the sender easier and decoding at the receiver easier. Network coding with xor operation in wireless mesh network has been studied in [15] which aims to improve network throughput. Optimal broadcast encoding decision problem which has been proved to be NP-hard is studied in [16]. Reliable broadcast using xor coding is studied in [17]. However, the encoding decision has not been introduced in [17].

Fountain codes (e.g. Raptor codes [18]) offer very low coding overhead and are (asymptotically) rate optimal when transmitting over erasure channels. However, decoding n packets is only possible after $n + \epsilon$ coded packets have been received and they do not consider the feedback information of receivers. Exactly the same problem addressed in the paper, how to best code repair packets, has also been addressed in [19], which studied the broadcast problem over independent erasure channels with perfect feedback and source coding using rate-optimal transmission schemes to optimize delay. The work in [20] addressed efficient feedback design to facitiate the retransmission process and the work in [21] aimed at minimizing decoding delay. A theoretical throughput upperbound of using network coding in retransmission was studied in [22] which assumed that the sender sent a packet and the acknowledgment for such a packet can be sent back to the sender at the same time slot. Different from the above work, we aim at the reduction of number of retransmissions and study the encoding decision based on the feedback from receivers.

III. System framework and Problem Description

In a wireless network with unreliable wireless communications, a receiver may not receive some data packets sent by the sender. Thus, a multicast session consists of sending original packets and encoded retransmission packets. Such two tasks can be interleaved with the following two modes:

- Sending a batch of original packets followed by sending one encoded retransmission packet where the batch size can be determined according to the estimated feedback delay. The time interval between two consecutive retransmission packets depends on how long it takes to receive the feedback from the receivers. A longer time interval can potentially utilize the gain of network coding better. This approach gives the sender enough time to collect feedback of recovered packets after the previous encoded retransmission packet is sent out. Thus, when it is scheduled to send an encoded retransmission packet, the encoding decision will be made based on the latest "wanted" packet set at receivers.
- Sending a batch of original packets followed by sending a batch of encoded retransmission packets. Under such a case, after sending a batch of original packets, with the feedback from the receivers, the source will determine a batch of encoded retransmission packets.

Let $R = \{r_1, r_2, \ldots, r_n\}$ be the set of receivers in a multicast session. Suppose that the set of original packets that the sender has sent out in the current transmission window is $P = \{p_1, p_2, \ldots, p_m\}$. Let $H(r_i) \subseteq P$ be the set of packets that receiver r_i has received successfully and $L(r_i) = P - H(r_i)$ be the set of packets that receiver r_i has lost. In this paper, we only consider XOR coding instead of linear network coding [23] since encoding and decoding operations using XOR is easy to be implemented with less overhead.

For the transmission mode of sending a batch of original packets followed by sending one encoded retransmission packet, our aim is to find an encoding decision such that the maximum number of receivers can decode out one "wanted" packet from their "wanted" packet sets where the "wanted" packet set at each receiver is dynamically updated. Such an encoding decision problem is referred to as *Dynamic Multicast Retransmission Encoding*(DMRE) problem in this paper.

For the transmission mode of sending a batch of original packets followed by sending a batch of retransmission packets, our aim is to find encoding decisions such that the minimum of number of retransmissions is needed to recover all packets in $L(r_i)$ for each $r_i \in R$. In this paper, we propose a cached based encoding decision where each receiver caches the received encoded retransmission packets even if it can not decode out one original packet in its "wanted" packet set currently until the end of current retransmission cycle. Such a problem is referred to as *Cache-based Multicast Retransmission Encoding* (CMRE) problem.

IV. DMRE

In this section, we first introduce an auxiliary graph $G(V, E)$ to model the problem. The encoding decision of DMRE is then converted to finding a maximum clique in the auxiliary graph. The gain of network coding in DMRE model is affected by wireless link reliability. In this paper, we use packet delivery ratio, which is the probability that a packet can be successfully received by a receiver, to characterize the wireless link reliability. We then analyze the impact of packet delivery ratio on network coding gain in DMRE model in this section.

A. Auxiliary Graph Construction and Algorithm

For each packet $p_j \in L(r_i)$, there is a corresponding vertex $v_{ij} \in V(G)$ where $1 \leq i \leq n$ and $1 \leq j \leq m$. $V_i = \{v_{ij} | p_j \in L(r_i), 1 \leq j \leq m\} \subseteq V(G)$ represents all lost packets at r_i. For any two different vertices $v_{i_1 j_1}, v_{i_2 j_2} \in V(G)$, there is a link $(v_{i_1 j_1}, v_{i_2 j_2}) \in E(G)$ if (1) $j_1 \neq j_2, p_{j_2} \in H(r_{i_1})$ and $p_{j_1} \in H(r_{i_2})$; or (2) $j_1 = j_2$.

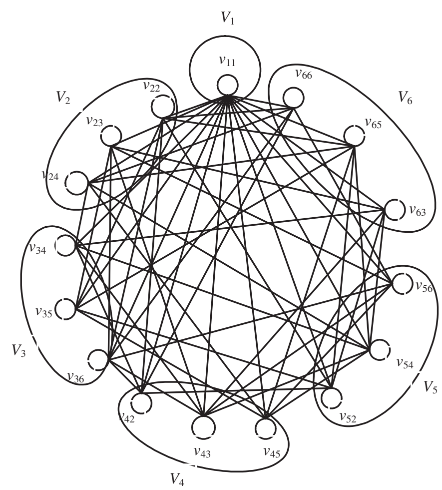


Fig. 2. The corresponding graph G of the example given in Fig. 1

Fig. 2 is the corresponding graph of the aforementioned example in section I. In Fig. 2, v_{22} represents that r_2 lost packet p_2, v_{36} represents that r_3 lost packet p_6, v_{42} represents that r_4 lost packet p_2. Since $p_2 \in H(r_3)$ and $p_6 \in H(r_2)$ in the aforementioned example in section I, there is a link (v_{22}, v_{36}). r_2 and r_4 lost the same packet p_2, so there is a link (v_{22}, v_{42}). According to the construction of G, we have the following lemma.

Lemma 1: For $e = (v_{i_1 j_1}, v_{i_2 j_2}) \in E(G)$, (1) if $j_1 = j_2$, r_{i_1} and r_{i_2} can recover p_{j_1} if s multicasts p_{j_1}; (2) if $j_1 \neq j_2$, r_{i_1} can recover p_{j_1} and r_{i_2} can recover p_{j_2} if s multicasts $p_{j_1} \oplus p_{j_2}$.

Proof: (1) is obvious. If $j_1 \neq j_2$, according to the construction of G, $e = (v_{i_1 j_1}, v_{i_2 j_2}) \in E(G)$ means that $p_{j_1} \in H(r_{i_2})$ and $p_{j_2} \in H(r_{i_1})$, thus, r_{i_1} can recover p_{j_1} by $p_{j_2} \oplus (p_{j_1} \oplus p_{j_2})$ and r_{i_2} can recover p_{j_2} by $p_{j_1} \oplus (p_{j_1} \oplus p_{j_2})$. Thus, (2) holds. ∎

We now discuss the general case of encoding when a clique can be found in G. Let $C = \{v_{i_1 j_1}, v_{i_2 j_2}, \ldots, v_{i_k j_k}\}$ be a clique in G. Let $P_C = \{p_j | v_{ij} \in C\}$, which is referred to as the lost packet set in C. Let $R_C = \{r_i | v_{ij} \in C\}$. Without loss of generality, let $P_C = \{p_{c_1}, p_{c_2}, \ldots, p_{c_t}\}$ where $t \leq k$. We have the following lemma.

Lemma 2: By multicasting $p' = p_{c_1} \oplus p_{c_2} \ldots \oplus p_{c_t}$, each $r_i \in R_C$ can recover packet p_j if $v_{ij} \in C$.

Proof: According to the construction of G, for each i_1, $r_{i_1} \in R_C$, there is only one vertex $v_{i_1 j_1} \in C$. Since C is a clique, r_{i_1} has all packets in P_c except p_{j_1} according to the link definition in G. Thus, r_{i_1} can recover p_{j_1} when receiving p'. ■

Lemma 2 indicates that one encoded retransmission packet can recover all lost packets in a clique $C \subseteq V(G)$. Since for each i_1, $r_{i_1} \in R_C$, there is only one vertex $v_{i_1 j_1} \in C$, the clique size of C equals to $|R_C|$ where $|R_C|$ is the cardinality of R_C. Thus, finding an encoded retransmission packet such that the maximum number of receivers can decode out one lost packet is equivalent to finding a maximum clique in the graph G.

DMRE problem aims to reduce the number of retransmissions by dynamically updating the "wanted" packet set of each receiver based on the feedback. For example, we can find a clique C=$\{v_{24}, v_{45}, v_{54}, v_{65}\}$ in G given in Fig. 2 and determine the first encoded retransmission packet to be $p_4 \oplus p_5$. If this encoded retransmission packet is received successfully, based on these feedback the sender can construct a new graph G_u which does not contain $\{v_{24}, v_{45}, v_{54}, v_{65}\}$ due to the packets recovered by using $p_4 \oplus p_5$, as Fig. 3 shows. Thus, based on the feedback information, we can find a maximum clique at each retransmission opportunity.

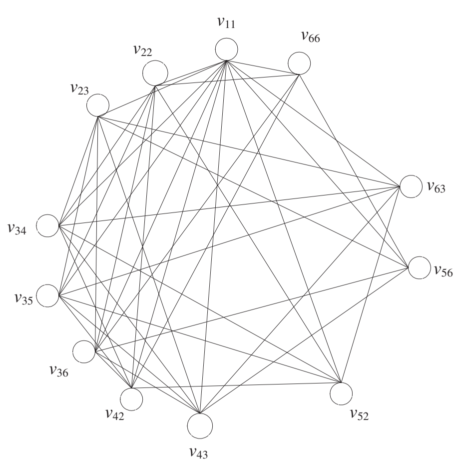


Fig. 3. The updated graph G_u

It has been shown that finding the optimal memoryless coding is NP-hard in [16]. We propose the following heuristic algorithm for DMRE problem. The algorithm will find a maximum clique C in updated graph G at first. Maximum clique problem is also NP-hard and there are many approximation algorithms. Due to the light computation capacity at wireless nodes, we propose to use greedy algorithm to find a maximum clique in a graph which adding vertices with the maximum degree into a clique until no larger clique can be found. We then identify P_C of C. The correspondent encoded retransmission packet is generated by xoring all packets in P_C. The pseudocode of the proposed algorithm for DMRE problem is given in Fig. 4.

1. Update $H(r_i)$ and $L(r_i)$ based on feedback, $1 \leq i \leq n$;
2. Construct a graph G based on $H(r_i)$ and $L(r_i)$, $1 \leq i \leq n$;
3. Find a maximum clique C in G;
4. Compute P_C for C;
5. $p' \leftarrow p_{c_1} \oplus p_{c_2} \oplus \ldots \oplus p_{c_k}$ where $p_{c_t} \in P_C$, $1 \leq t \leq k$;
6. Send p' to receivers;

Fig. 4. Algorithm for DMRE problem

B. The impact of packet delivery ratio on network coding gain

If the "wanted" packet sets at all receivers are the same, there is no gain of using network coding compared with traditional reliable multicast which retransmits original packets. For any two lost packets p_{j_1}, p_{j_2}, the gain of network coding is obtained only when there exists at least two receivers r_{i_1}, r_{i_2} such that there is a link between v_{i_1,j_1} and v_{i_2,j_2} in G.

Let $W(p_j) = \{r_i | p_j \in L(r_i)\}$ be the set of receivers which have lost p_j. For any two lost packets p_{j_1}, p_{j_2}, let us consider the probability of $W(p_{j_1}) \cap W(p_{j_2}) = \emptyset$ which we denote as θ_e. If θ_e is low, many receivers lost both p_{j_1} and p_{j_2}. As a result, most likely we need to retransmit p_{j_1} and p_{j_2} separately as two retransmission packets. If θ_e is high, there are many receivers who only lost either p_{j_1} or p_{j_2}, where potentially we only need to retransmit $p_{j_1} \oplus p_{j_2}$. Thus, the network coding gain is larger. In this section, we analyze the impact of packet delivery ratio on θ_e, which demonstrates the impact of packet delivery ratio on the gain of network coding. Let the probability that receiver r_i in a multicast session can receive a packet successfully be ρ_i. For any two lost packets p_{j_1} and p_{j_2}, let θ_e be the probability of $W(p_{j_1}) \cap W(p_{j_2}) = \emptyset$. We have

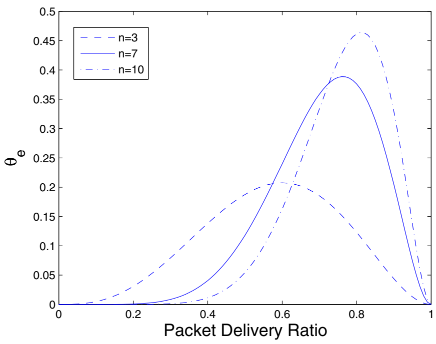
Theorem 1: $\theta_e = \prod_{i=1}^{n} (2\rho_i - \rho_i^2) - 2\prod_{i=1}^{n} \rho_i + \prod_{i=1}^{n} \rho_i^2$, where n is the number of receivers.

Proof: If $W(p_{j_1}) \cap W(p_{j_2}) = \emptyset$, then any receiver r_i will not lose both p_{j_1} and p_{j_2}. Consider receiver r_i, since the probability that r_i receives a packet successfully is ρ_i, then the probability that r_i loses both p_{j_1} and p_{j_2} is $(1 - \rho_i)^2$. Thus the probability that r_i will not lose both p_{j_1} and p_{j_2} is $1 - (1 - \rho_i)^2$. Due to the independence of packet loss at every receiver, $P\{W(p_{j_1}) \cap W(p_{j_2}) = \emptyset\} = \prod_{i=1}^{n} (1 - (1 - \rho_i)^2)$. Since p_{j_1}, p_{j_2} are lost packets, $W(p_{j_1}) \neq \emptyset, W(p_{j_2}) \neq \emptyset$, then we can have:

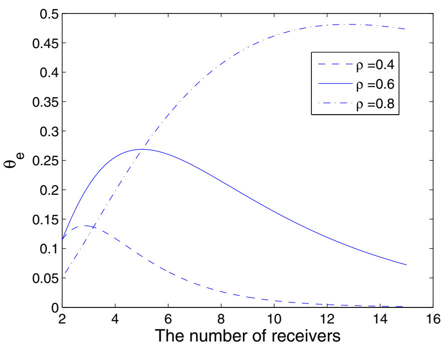
$$\begin{aligned}
\theta_e &= P\{W(p_{j_1}) \cap W(p_{j_2}) = \emptyset, W(p_{j_1}) \neq \emptyset, W(p_{j_2}) \neq \emptyset\} \\
&= P\{W(p_{j_1}) \cap W(p_{j_2}) = \emptyset\} - P\{W(p_{j_1}) = \emptyset\} \\
&\quad - P\{W(p_{j_2}) = \emptyset\} + P\{W(p_{j_1}) = \emptyset, W(p_{j_2}) = \emptyset\} \\
&= \prod_{i=1}^{n} (1 - (1 - \rho_i)^2) - \prod_{i=1}^{n} \rho_i - \prod_{i=1}^{n} \rho_i + \prod_{i=1}^{n} \rho_i^2 \\
&= \prod_{i=1}^{n} (2\rho_i - \rho_i^2) - 2\prod_{i=1}^{n} \rho_i + \prod_{i=1}^{n} \rho_i^2.
\end{aligned}$$

■

Based on the above theorem, we can get the following corollary.



(a) θ_e with the increase of ρ



(b) θ_e with the increase of n

Fig. 5. Probability curves of θ_e estimating the coding gain

Corollary 1: Let the probability that receivers in a multicast session can receive a packet successfully be the same, denoted by ρ. For any two lost packets p_{j_1} and p_{j_2}, let θ_e be the probability of $W(p_{j_1}) \cap W(p_{j_2}) = \emptyset$. We have $\theta_e = (2\rho - \rho^2)^n - 2\rho^n + \rho^{2n}$, where n is the number of receivers.

Fig. 5 depicts the curves of probability θ_e with the increase of packet delivery ratio and the number of receivers given that all receivers have the same packet delivery ratio. The curves of θ_e obtained by simulation are consistent with the analysis given in Fig. 5. The simulation results in Section VII also shows that the change of network coding gain under different packet delivery ratio is consistent with the change of θ_e under different packet delivery ratio, which further demonstrates that θ_e can be used to estimate the network coding gain indeed.

V. CMRE

The work above is based on the assumption that memoryless decoding is conducted at the receivers, i.e. once the encoded packet p' arrives at receiver r_i, if r_i cannot decode p' immediately, r_i just drops packet p'. However, if we allow receivers to store packets which cannot be decoded immediately, we can further reduce the number of retransmissions. Our CMRE model is that receivers store packets which cannot be decoded immediately, and decode until enough encoded retransmission packets are received.

```
l ← 0;
P′ ← ∅;
while (V(G) ≠ ∅ ){
  p′_l ← 0;
  Q(p′_l) ← ∅;
  for (i← 1 to n){
    j_m ← min{j|v_ij ∈ V_i};
    if  (p_{j_m} ∉ Q(p′_l)){
      p′_l ← p′_l ⊕ p_{j_m};
      Q(p′_l) ← Q(p′_l) ∪ {p_{j_m}};
    }
    delete v_{ij_m} in G;
  }
  P′ ← P′ ∪ {p′_l};
  l ← l + 1.
}
```

Fig. 6. Initial partial solution construction algorithm for CMRE problem

In what follows in this section, we introduce the encoding strategy at the source node at first followed by the decoding procedure at the receivers.

A. Encoding Algorithm

We propose an encoding algorithm which aims to use the minimum number of retransmissions to recover all lost packets at the receivers. Let $l_{max} = max_{r_i \in R}|L(r_i)|$, where $|L(r_i)|$ is the cardinality of $L(r_i)$. Because a receiver can at most recover one original packet upon receiving an encoded (xor) packet, any encoding algorithm needs at least l_{max} retransmission packets. Therefore, l_{max} is the lower bound of the number of retransmissions even under the ideal case that all retransmission packets are successfully received by all receivers. The lower bound of the expected number of retransmissions considering the loss of retransmission packets will be derived in the next section. The proposed encoding algorithm works as two stages. At the first stage, we construct l_{max} encoded packets. Each encoded packet is the xor of some original packets, of which, at most one is selected from the lost packets of a receiver. With these l_{max} encoded packets, some receivers may not recover all lost packets, then at the second stage we need to append more necessary packets to ensure all receivers can recover all lost packets.

At the first stage, we construct l_{max} encoded retransmission packets. According to the definition of G in Section IV, for any $i, 1 \le i \le n$, if $j \ne k$, $(v_{ij}, v_{ik}) \notin E(G)$. We can partition V into n subsets $\{V_1, V_2, \ldots, V_n\}$ where $V_i = \{v_{ij} | p_j \in L(r_i), 1 \le j \le m\}$ is the set of lost packets of receiver r_i. The main idea of our construction algorithm is that we select a lost packet corresponding to a vertex in every V_i and xor them together as one encoded retransmission packet. For every V_i, we select v_{ij_m}, $j_m = min\{j | v_{ij} \in V_i\}$, after encoded packet being decided we delete v_{ij_m} in G. At the end of the construction algorithm, we get l_{max} encoded retransmission packets. This stage constructs a partial solution for CMRE problem. The initial partial solution construction algorithm is given in Fig. 6.

For example, suppose that sender s has sent packets $\{p_1,p_2,p_3,p_4,p_5,p_6\}$, r_1 has lost $\{p_1,p_2\}$, r_2 has lost $\{p_1,p_3\}$, r_3 has lost $\{p_1,p_4\}$, r_4 has lost $\{p_2,p_3\}$, r_5 has lost $\{p_2,p_4\}$, and r_6 has lost $\{p_3,p_4\}$. From the definition of graph G, we get the vertex subsets $V_1 = \{v_{11}, v_{12}\}$, $V_2 = \{v_{21}, v_{23}\}$, $V_3 = \{v_{31}, v_{34}\}$, $V_4 = \{v_{42}, v_{43}\}$, $V_5 = \{v_{52}, v_{54}\}$, $V_6 = \{v_{63}, v_{64}\}$. The initial partial solution is $P' = \{p_1 \oplus p_2 \oplus p_3, p_2 \oplus p_3 \oplus p_4\}$.

Given P' constructed at the first stage, we need to find that whether P' is enough and what are needed for all receivers to recover all lost packets. At the second stage, we append more necessary retransmission packets based on the initial partial solution such that all receivers can recover their lost packets.

For receiver r_i, when it receives an encoded retransmission packet p'_k, let P'_k be the set of packets used to encode p'_k, we define a *receiving vector* $v_{ik} = (a_1, a_2, \ldots, a_m)$, $a_j = 1$ if $p_j \in L(r_i)$ and $p_j \in P'_k$, else a_j=0, $1 \le i \le n, 1 \le j \le m$.

Assume that r_i has lost k packets and received l retransmission packets, $l \ge k$. Let $L(r_i) = \{p_{i_1}, p_{i_2}, \ldots, p_{i_k}\}$. r_i can construct a matrix $M^i_{l\times m}$ based on receiving vectors. If $p_j \in L(r_i)$, we keep the j-th column in $M^i_{l\times m}$, otherwise, we remove it from $M^i_{l\times m}$. After such transformation, we get a $l \times k$ sub-matrix $M^i_{l\times k}$.

The following transformation can find out whether the j-th lost packet at r_i can be recovered or not. We apply Gaussian elimination on $M^i_{l\times k}$, if $l > k$, there exists some all-zero rows. After randomly deleting some all-zero rows, we can transform $M^i_{l\times k}$ from a $l\times k$ matrix to a $k\times k$ upper triangular matrix $M^i_{k\times k} = (a_{tj})_{k\times k}$. Based on $M^i_{k\times k}$, considering the set $J = \{j | a_{tj}$ is the first non-zero element of row $t\}$. From the following lemma, we know that if $j_1 \notin J, 1 \le j_1 \le k$, the j_1-th lost packet at r_i can not be recovered.

Lemma 3: Consider n variables $x_1, x_2, \ldots, x_n$ and n equations, $M_{n\times n}$ is the coefficient matrix. After Gaussian elimination on M, $J = \{j|a_{tj}$ is the first non-zero element of row $t\}$, if $j_1 \notin J$, we can not solve for x_{j_1}.

Proof: After the Gaussian elimination, M is transformed to an upper triangular matrix M'. $J = \{j|a_{tj}$ is the first non-zero element of row $t\}$, if $j_1 \notin J$, then a_{tj_1} is not the first non-zero element of row t, $1 \le t \le n$. That means $a_{j_1 j_1} = 0$ if M' is an upper triangular matrix.

$$\begin{pmatrix} \ddots & & & & \\ & a_{j_1-1,j_1-1} & & & \\ & & 0 & a_{j_1,j_1+1} & \\ & & & 0 & \\ 0 & & & & \ddots \end{pmatrix}$$

According to the upper triangular matrix M', we can not solve for x_{j_1}.

■

With Lemma 3, we can easily identify which packet of $L(r_i)$ can not be recovered and determine the necessary encoded retransmission packets. If $j_1 \notin J, 1 \le j_1 \le k$, $p_{i_{j_1}}$ will be sent as a retransmission packet from the sender. The appending process of the second stage is given in Fig. 7.

$Q' \leftarrow \emptyset$;
for ($i \leftarrow 1$ to n) **do**
  $l \leftarrow |P'|$;
  $k \leftarrow |L(r_i)|$; ($L(r_i) = \{p_{i_1}, p_{i_2}, \ldots, p_{i_k}\}$)
  Construct matrix $M^i_{l\times m}$ and $M^i_{l\times k}$ based on receiving vectors of r_i;
  Do Gaussian elimination on $M^i_{l\times k}$ to make an upper triangular matrix $M^i_{k\times k} = (a_{tj})_{k\times k}$;
  $J \leftarrow \{j|a_{tj}$ is the first non-zero element of row $t\}$;
  for ($j \leftarrow 1$ to k) **do**
    if ($j \notin J$) **then**
      $Q' \leftarrow Q' \cup \{p_{i_j}\}$;
      break;

Fig. 7. Appending algorithm for CMRE problem

Combining results of the first stage and the second stage, we can get the encoded retransmission packets from $P' \cup Q'$, where P' is the set of encoded packets from stage 1 and Q' is the set of original packets from stage 2. For the example above, at the second stage, based on P' we can find out that all receivers can recover their lost packets but r_4. Based on the receiving vectors of r_4, $(0,1,1,0)$,$(0,1,1,0)$, r_4 can construct matrix M

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

After Gaussian elimination, we get the upper triangular matrix

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

We find that $J = \{1\}$ and $2 \notin J$. Because $L(r_4) = \{p_2, p_3\}$, then $Q' = \{p_3\}$. Finally we get the set of retransmission packets $P' \cup Q' = \{p_1 \oplus p_2 \oplus p_3, p_2 \oplus p_3 \oplus p_4, p_3\}$.

B. Decoding Algorithm

Similarly, each receiver can determine how to decode each lost packet by the operations of Gauss-Jordan elimination. Upon receiving l encoded retransmission packets and with its received original data packets, each receiver can construct a matrix as follows.

- Suppose that a receiver r_i receives l encoded retransmission packets, denoted by $p'_1, p'_2, \ldots, p'_l$. For each p'_j, the packet head will specify which original packets are encoded together, P'_j denotes the original packets set used to encode p'_j. With such packet head information, we can construct a m-dimension vector $v = (a_1, a_2, \ldots, a_m)$ where a_k is 1 if $p_k \in P'_j$, otherwise a_k is 0.
- For each received original data packet p_j where $p_j \in H(r_i)$, we can also construct a m-dimension vector $v = (0, \ldots, a_j, \ldots, 0)$ where only $a_j = 1$.

As a result, r_i constructs a $(l+|H(r_i)|)\times m$ matrix M, where $|H(r_i)|$ is the cardinality of $H(r_i)$. For each row of M, we maintain the correspondent packet information in an array B. For example, if the vector correspondent to p'_j is row k_1 of M, we set $B[k_1]=p'_j$. Similarly, if the vector correspondent to $p_j \in H(r_i)$ is row k_2 of M, we set $B[k_2]=p_j$.

We then conduct Gauss-Jordan elimination on M to transform M to an $m \times m$ diagonal matrix by xoring rows and exchanging rows. We record the correspondent decoding operation by changing the packet information in B correspondingly.

- If we xor row j_1 to row j_2, then $B[j_2]=B[j_2]\oplus B[j_1]$.
- If we exchange row j_1 with row j_2, we exchange the information in $B[i_1]$ with the information in $B[i_2]$.

After M is transformed to an $m \times m$ diagonal matrix, $B[j]$ records how to decode and recover lost packet p_j. For the example given in Section V-A, the set of encoded retransmission packets is $\{p'_1,p'_2,p'_3\}$, $p'_1 = p_1 \oplus p_2 \oplus p_3$, $p'_2 = p_2 \oplus p_3 \oplus p_4$, $p'_3 = p_3$, $L(r_1)=\{p_1,p_2\}$. After r_1 has received p'_1,p'_2,p'_3, it can construct M and B as follows:

$$M = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, B = \begin{pmatrix} p_3 \\ p_4 \\ p'_1 \\ p'_2 \\ p'_3 \end{pmatrix}$$

After the Gauss-Jordan elimination,

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} p'_1 \oplus p'_2 \oplus p_4 \\ p_3 \oplus p'_2 \oplus p_4 \\ p_3 \\ p_4 \end{pmatrix}$$

From the information in B, we know that r_1 can recover p_1 according to $p'_1 \oplus p'_2 \oplus p_4$, and recover p_2 according to $p_3 \oplus p'_2 \oplus p_4$.

After such a batch of retransmission packets are sent out, if some receivers still can not recover some packets in their "wanted" packet sets due to the loss of retransmission packets, they will send feedbacks to the source. The source can determine either to send a new batch of original packets and recover the lost packets in the next round or immediately trigger a new round of retransmissions. Such a decision can be made based on the available buffer size at receivers.

VI. Performance Analysis

Suppose that a batch of m original packets are sent, we analyze the lower bound of the expected number of retransmissions in both DMRE model and CMRE model. In deriving such a lower bound, we also take the loss of retransmission packets into consideration. Such a lower bound shows that the maximum gain of using network coding compared with the traditional reliable multicast. We assume that the probability that the receiver r_i in a multicast session can receive a packet successfully is ρ_i.

Theorem 2: The lower bound of the expected number of retransmissions using network coding is

$$N_c = \frac{m(1-\min_{1\le i\le n}\{\rho_i\})}{\min_{1\le i\le n}\{\rho_i\}}$$

where $\rho_i, 1 \le i \le n$ is the packet delivery ratio and n is the number of receivers.

Proof: After the sender sending out m original packets, due to packet loss, the expected number of packets received successfully at receiver r_i is $m\rho_i$. Considering receiver r_j where $\rho_j = min\{\rho_i, 1 \le i \le n\}$, in the ideal case, if r_j can recover all "wanted" packets from encoded retransmission packets, other receivers may also be able to recover their "wanted" packets from those encoded retransmission packets. Thus, the lower bound of the number of retransmissions will be the expected number of retransmissions to recover all "wanted" packets of r_j.

Since r_j may lose $m(1-\rho_j)$ packets in average and r_j can only decode out at most one "wanted" packet from each encoded retransmission packet, the sender needs to send at least $m(1-\rho_j)$ encoded packets to r_j successfully. Thus, the lower bound of the number of retransmissions is the expected number of transmission sending $m(1-\rho_j)$ packets to r_j successfully. The number of transmission attempts before receiver r_j successfully receives a packet follows the geometric distribution with parameter ρ_j, which is, the average number of transmissions before receiver r_j can successfully receive a packet is $\frac{1}{\rho_j}$. Thus, the total number of transmissions sending $m(1-\rho_j)$ packets to r_j successfully is $\frac{m(1-\rho_j)}{\rho_j}$. Since $\rho_j = \min_i\{\rho_i\}$, we can get the lower bound of the expected number of retransmissions using coding is

$$N_c = \frac{m(1-\min_{1\le i\le n}\{\rho_i\})}{\min_{1\le i\le n}\{\rho_i\}}.$$

■

Consider the packet header requirements for our retransmission scheme. It identifies the native packets XOR-ed in the encode packet. In the schemes we propose, arbitrary combinations of previously send packets could be XOR-ed and then resent. If the retransmission batch size is m, then to send the retransmission encoded packets, it would normally need $log(m)$ bits for the packet header. It can be easily made negligible compared with the data packet sizes.

VII. Simulation

In this section, we demonstrate the effectiveness of our retransmission schemes through simulation using C++. In our experiment, we group the packets generalized by the sender into batches, and each batch has m packets. Retransmission process starts after every m packets are transmitted. We set $m = 20$ as in [5].

In order to simulate packet loss at the receiver side, we use packet delivery ratio ρ to randomly generate the "wanted" set $L(r_i)$ at each receiver r_i. Each packet p_j $(1 \le j \le m)$ can be received at r_i successfully with probability ρ and be lost at r_i with probability $1-\rho$. In the simulation, we will study the case where ρ is set to be the same for all receivers and the case where different receivers have different ρ's.

In the simulation, we are interested in the network coding gain from different perspectives, e.g. the average number of retransmissions, the number of reduced retransmissions, and the retransmission reduction ratio. We study the impacts of ρ, batch size m and the number of receivers n on the network coding gain under different network coding schemes. We also compare the performance of our proposed algorithms with the derived lower bound of the expected number of retransmissions. For each simulation setting, we simulate 200 instances and report the average performance.

A. The impact of ρ on network coding gain

First we evaluate the average number of retransmissions without coding and with memoryless coding. The results of CMRE which is not a memoryless coding will be mentioned later. For memoryless coding, we compare the performance of minimum clique partition based coding [5] with our proposed DMRE.

With regard to the impact of ρ on network coding gain, we only report the case when ρ is the same for all receivers. When ρ is different for different receivers, similar impact on network coding gain can be observed and the results are omitted in this paper to avoid the redundancy.

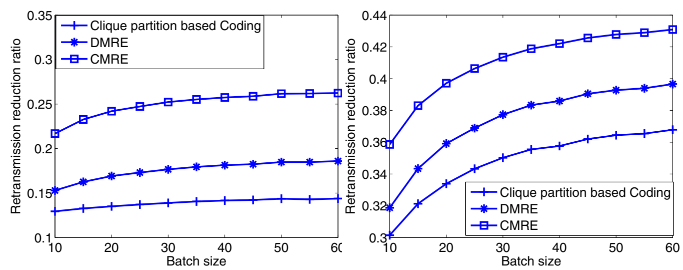
Fig. 8 shows the average number of retransmissions with increasing ρ. From Fig. 8, for all cases of $n = 3, 7$ and 10, the average number of retransmissions in DMRE scheme is less than that in minimum clique partition scheme and the latter is less than the average number of retransmissions without network coding. With increasing ρ, the average number of retransmissions decreases for all three schemes, since less packets are lost.

Fig. 9 depicts the number of reduced retransmissions of minimum clique based coding scheme and our proposed DMRE scheme compared with the case without network coding. When ρ is neither too large nor too small, the number of reduced retransmissions using DMRE is 10% more than minimum clique based coding scheme on average. From Fig. 9(a), when $\rho = 0.6$, the network coding gain reaches the maximum which is consistent with our analysis in Fig. 5(a). It is the same for the case of $n = 7$ in Fig. 9(b) and the case of $n = 10$ in Fig. 9(c). Fig. 9 shows that when ρ is too small or too large, the network coding gain is marginal. It can be explained as follows. If ρ is too small, each receiver almost lost all packets, which means that the lost packets at receivers are almost the same. Thus, almost each original data packet needs to be retransmitted. Therefore, the network coding gain is marginal. If ρ is too large, the number of lost packets at receivers is small. The number of retransmissions is already very small even without coding. So the network coding gain is marginal.

B. The impacts of batch size and the number of receivers on the network coding gain

In order to study the impacts of batch size m and the number of receivers n on the network coding gain, we use the retransmission reduction ratio $\beta = \frac{Num_{nocoding} - Num_{coding}}{Num_{nocoding}}$ as performance metric where $Num_{nocoding}$ is the number of retransmissions without network coding, and Num_{coding} is the number of retransmissions with network coding. We compare retransmission reduction ratio of minimum clique partition based scheme with DMRE and CMRE schemes.

With regard to the impacts of batch size m and the number of receivers n on the network coding gain, similar impacts can be observed for both the case when ρ is the same for all receivers and the case when ρ is different for different receivers. We only report the case when ρ is different for different receivers in this paper to avoid redundancy.



(a) ρ in $[0.2, 1]$ (b) ρ in $[0.5, 1]$

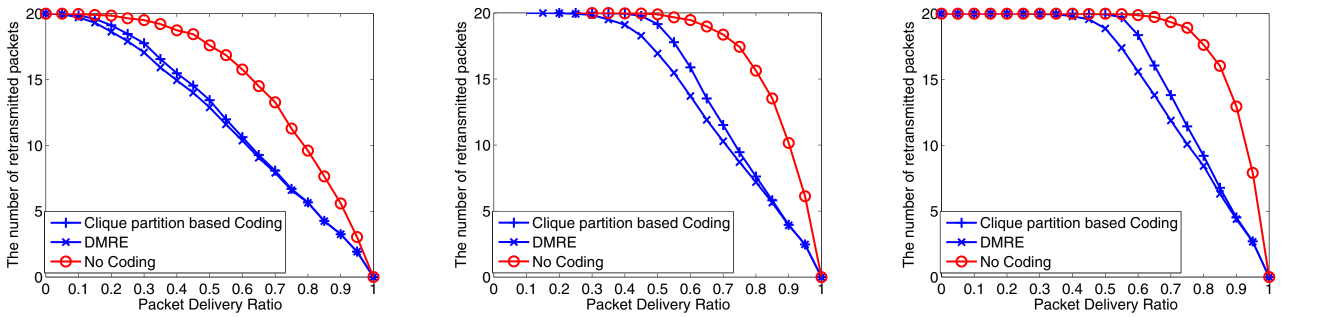
Fig. 10. Retransmission reduction ratio vs. batch size

Fig. 10 shows the impact of batch size m on the network coding gain which is measured by the retransmission reduction ratio for $n = 5$. In Fig. 10(a), ρ is uniformly distributed in $[0.2, 1]$. In Fig. 10(b), ρ is uniformly distributed in $[0.5, 1]$. As shown in Fig. 10, for fixed ρ, the probability that a lost packet of a receiver is successfully received by another receiver and vice versa increases with increasing m. The reason is that the received packets at each receiver will be randomly distributed in a more larger set of original data packets while m increases. So the network coding gain becomes larger. Due to the same reason, comparing the results in Fig. 10(a) with the results in Fig. 10(b), when ρ becomes larger, with increasing m, the network coding gain increases faster.

Fig. 11 shows the impact of the number of receivers n on the network coding gain. We can see that the retransmission reduction ratio increased first and then decreased with increasing n. This can be explained according to the analysis result in Section IV-B and Fig. 5(b). The probability that a lost packet of some receivers is successfully received by other receivers and vice versa is related to both ρ and n. Given ρ, the probability increased first and then decreased with increasing n. Thus, the network coding gain also increased first and then decreased with increasing n.

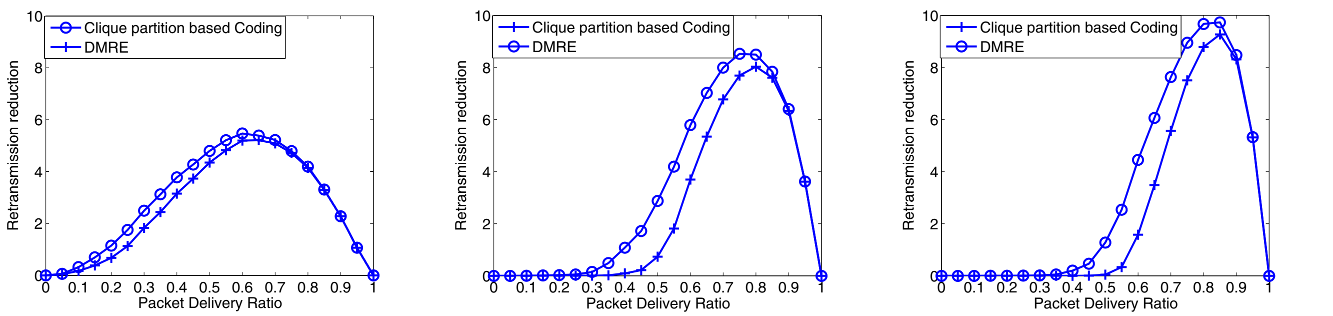
C. Comparison with the derived lower bound of the expected number of retransmissions

We now compare the gain of network coding using DMRE and CMRE with the derived lower bound of the expected number of retransmissions in Theorem 2. We



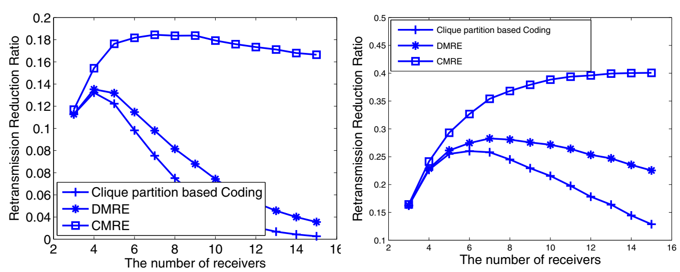
(a) $n = 3$ (b) $n = 7$ (c) $n = 10$

Fig. 8. Average number of retransmission vs. ρ when $m = 20$



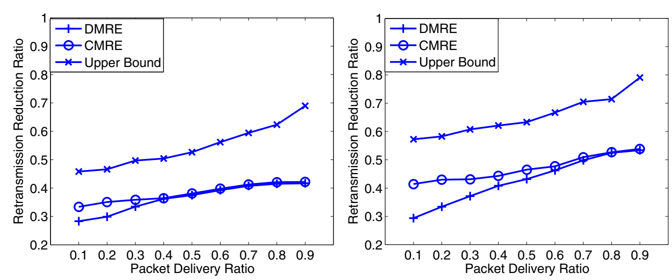
(a) $n = 3$ (b) $n = 7$ (c) $n = 10$

Fig. 9. Retransmission reduction vs. ρ when $m = 20$



(a) ρ in $[0.2, 1]$ (b) ρ in $[0.5, 1]$

Fig. 11. Retransmission reduction ratio vs. the number of receivers



(a) $n = 3$ (b) $n = 5$

Fig. 12. Retransmission reduction ratio vs. ρ when $m = 20$

report the results in terms of the retransmission reduction ratio $\beta = \frac{Num_{nocoding} - Num_{coding}}{Num_{nocoding}}$. From [17] we can get the theoretical result of $Num_{nocoing}$, $N_n = m \sum_{i_1, i_2, \ldots, i_n} \frac{(-1)^{i_1+i_2+\ldots,i_n-1}}{1-(1-\rho)^{i_1+i_2+\ldots+i_n}} - m$, where $i_1, i_2, \ldots, i_n \in \{0, 1\}$. Under the performance metric of retransmission reduction ratio, given the lower bound of the expected number of retransmissions N_c derived in Theorem 2 and the theoretical result of $Num_{nocoding}, N_n$, we can calculate the upper bound of retransmission reduction ratio $\beta_O = 1 - \frac{N_c}{N_n}$.

Fig. 12 shows the network coding gain when batch size $m = 20$. From Fig. 12, we can see that using coding can reduce about 40% of retransmissions. CMRE performs better than DMRE and the number of retransmissions using coding methods are upper bounded.

VIII. CONCLUSION

Using network coding to reduce the number of retransmissions for reliable multicast in wireless LAN has been studied in [5]. In this paper, we propose two new models to further reduce the number of retransmissions for reliable multicast with network coding. The first model is a memory-less model which minimizes the number of retransmissions by updating the available packets at each receiver after sending each encoded retransmission packet. Such a model is referred to as *Dynamic Multicast Retransmission Encoding* (DMRE). The other model requires buffer at the receiver side where a receiver will buffer all received encoded retransmission packets and decode them when enough encoded packets are received. Such a model is referred to as *Cache-based Multicast Retransmission Encoding*(CMRE) model. Since each receiver can "accumulate" information for decoding in the CMRE model, it can be expected that CMRE model can recover lost packets with less number of retransmission than that in DMRE model. However, CMRE model requires larger buffer size than that in DMRE model since each receiver also needs to buffer its received encoded retransmission packets. The problem to minimize the number of retransmissions under both DMRE and CMRE models are NP-hard. Effective heuristic algorithms are proposed in this paper. The impacts of packet delivery ratio and the number of receivers on the network coding gain are analyzed. Our simulation results show that the network coding gain of CMRE model excels DMRE model which is consistently better than the minimum clique partition based coding scheme. The simulation results also show that the impacts of packet delivery ratio and the number of receivers on the network coding gain are consistent with our theoretical analysis.

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