

# Interference and Power Constrained Broadcasting and Multicasting in Wireless Ad Hoc Networks with Directional Antennas

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**Abstract**—Broadcasting/Multicasting problems have been well studied in wireless ad hoc networks. However, only a few approaches take into account the low interference and energy efficiency as the optimization objective simultaneously. In this paper, we study the interference and power constrained broadcast/multicast and the delay-bounded interference and power constrained broadcast/multicast routing problems in wireless ad hoc networks using directional antennas. We propose an approximation and a heuristic algorithm for the two problems, respectively. Importantly, motivated by the study of above optimization problems, we propose approximation schemes for two multi-constrained directed Steiner tree problems, respectively. Broadcast/Multicast message by using the trees found by our algorithms tend to have less channel collisions and higher network throughput. The theoretical results are corroborated by simulation studies.

**Keywords:** wireless ad hoc networks, directional antennas, broadcasting/multicasting, energy efficient, interference, approximation algorithm.

## I. INTRODUCTION

As a promising technology for many application domains, wireless ad hoc networks have received significant attention in recent years [1]. Wireless ad hoc network consists of a collection of communication nodes dynamically forming a temporary network without the use of any existing network infrastructure. In such networks, each node is powered by batteries that may not be possible to be recharged or replaced during a mission. Consequently, the limited energy makes the energy efficiency become one of the primary issues in such networks [1].

Broadcasting/multicasting are important functions in applications of wireless ad hoc networks, such as cooperative operations, data dissemination, routing discover, and so on. There have been a lot of works on energy efficient broadcast/multicast routing problem [2], but only a few methods take into account the low interference as the optimization objective. The impacts of interference in wireless multihop networks have been observed and studied both theoretically and empirically in the literatures [3]. When a network node

transmit data to another node over a wireless channel, due to possible channel interference, the transmitted data may be corrupted in transit and data must repeatedly retransmit until it is received correctly at the terminal. The data retransmission may result in energy waste. Therefore, a good broadcast/multicast algorithm should be aware of interference as well as power consumption.

With the emergency of the smart directional antennas technology, using the directional antennas to improve network performance becomes more and more popular in recent years. Previous researches have shown that the application of directional antennas can further reduce the energy consumption and the radio interference [4], [5]. As a result, the use of directional antennas has a great potential in wireless ad hoc networks. There are two techniques used in directional antenna systems: *switched beam* and *steerable beam* [5]. This paper uses the *Switched beam* directional antennas model, which divides the transmission range of each node into different sectors. Each node can switch on one or several sectors for transmitting.

Energy efficiency and low interference are the two most important factors for improving the performance of wireless ad hoc networks. In this paper, we study the interference and power constrained broadcast/multicast routing problem in wireless ad hoc networks using directional antennas. Note that the directional characteristic discussed in this paper is from the point of view of transmitting, but not from receiving.

The contributions of this paper can be summarized as follows: We study two new broadcasting/multicasting problems:

- *Interference and Power Constrained Broadcasting/Multicasting with Directional antennas problem (D-IPCB/IPCM).*
- *Delay-Bounded Interference and Power Constrained Broadcasting/Multicasting with Directional antennas problem (DB-D-IPCB/IPCM).*

Both problems are NP-hard. Importantly, motivated by the study of above optimization problems, we first study the multi-constrained directed Steiner tree (MCST) and delay-bounded multi-constrained directed Steiner tree (DB-MCST) problems. And propose an approximation scheme for the MCST problem

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and DB-MCST problem, respectively. These two approximation schemes can be used to solve some important optimization problems.

Based on the algorithms for the MCST problem and DB-MCST problem, we present an approximation algorithm and a efficient heuristic for D-IPCB/IPCM and DB-D-IPCB/IPCM problem, respectively. Broadcast/Multicast routing by means of our algorithms can dramatically reduce channel collision and improve the network throughput. In, addition, the trade-off between total power and total interference of a broadcast/multicasting tree found by our algorithms is guaranteed to be controllable by given tolerance values. By carefully setting these values, broadcasting/multicasting trees obtained will not only be energy-efficient, but also have low interference.

To our best knowledge, this is the first paper which jointly considers low-interference and energy efficient issues for broadcasting/multicasting problems in wireless ad hoc networks with directional antennas. The two approximation schemes we proposed in this paper can efficiently give solutions for the multi-constrained Steiner tree problem (or the delay-bounded one) with theoretical performance guarantee.

The rest of the paper is organized as follows: In Section II, we briefly summarize some related works. Section III describes the network, directional antennas and interference models, and then we formulate the optimization problems. Several approximation algorithms and heuristics for the D-IPCB/IPCM and DB-D-IPCB/IPCM problems are proposed in Section IV and V. Performance evaluations and simulation results about proposed algorithms are exhibited in Section VI. Finally, we give a conclusion in Section VII.

## II. RELATED WORKS

Broadcasting and multicasting are the important functions in applications of wireless ad hoc networks. There have been a lot of works on energy efficient broadcast/multicast routing [6]–[13]. However, most of these works do not take into account the low interference energy efficiency as optimization objectives simultaneously.

There are some works studying the interference issue. In [3], Jain *et al.* considered the issue of interference when calculating the maximum throughput by a wireless network. They showed that a key issue of impacting performance is wireless interference between neighboring nodes. In other words, by employing interference-aware routing protocols, there is opportunity for achieving throughput gains. Burkhart *et al.* [14] first raised a fundamental question "Does topology control reduce interference?". They showed that traditional topology control methods will not always produce a sub-graph whose interference is within a constant factor of the optimum. They also gave a concise and intuitive definition of interference. Tang *et al.* [15] studied the interference-aware unicast routing problems in wireless multi-hop networks using directional antennas with dynamic traffics. They gave new definition of link interference and proposed several interference-aware routing algorithms. Li *et al.* [16] studied the interference-aware topology control problems for wireless

sensor networks. Ref. [17], [18] discussed the distributed construction of network topology with low interference and energy efficiency simultaneously.

Khuller *et al.* in [19] first introduced the  $(\alpha, \beta)$ -tree problem in wired networks and proved that there is a trade-off between SPT and MST. Based on the  $(\alpha, \beta)$ -tree, Li *et al.* [20] studied the construction of delay-bounded strongly connected broadcast arborescence problem and proposed an approximation algorithm.

However, none of works addressed broadcast/multicast routing problem with directional antennas taking into account low interference and energy efficiency simultaneously. In this paper, we study the interference and power constrained broadcast/multicast routing problem in wireless ad hoc networks using directional antennas.

## III. PRELIMINARIES

In this section, we first introduce the *switched beam* directional antennas and network model. Then, we give the interference model and formally formulate the D-IPCB/IPCM and DB-D-IPCB/IPCM problems.

### A. Network and Directional Antenna Models

We assume that there are  $n$  sensor nodes  $v_1, v_2, \dots, v_n$  deployed in the 2-D Euclidean plane with known positions, and all nodes have distinctive identities and each node  $v_i$  is equipped with a *switch beam* directional antenna and has  $l(i)$  transmission *directions* (i.e. the antenna sectors). We also assume that the directional antenna device of each node can switch on one or several sectors for transmission. We use a general antennas model with fewer assumptions than existing models [9], [11]. We assume each node is equipped with a *switched beam* directional antenna, and different sectors may be irregular, overlapping and have different shapes. The only constraint is that each antenna sector must have a fixed size and shape. This is more practical in real systems because the radio performance may degenerate dramatically, i.e. although the antenna sectors are identical in idealizing model, their transmission ranges may become very different in practice. The network can be modeled by a directed graph  $G = (V, E, c, f, d)$ , where  $V$  is the set of nodes and  $E$  is the set of communication link. There is a directed edge  $(v_i, v_k)$  in  $E$  if node  $v_k$  is within the transmission range of a  $v_i$ 's switched-on *direction*. The cost functions  $c(v_i, v_k)$ ,  $f(v_i, v_k)$  and  $d(v_i, v_k)$  represent the power cost, interference value and transmission delay when node  $v_i$  transmit data to  $v_k$ , respectively. We also assume the nodes in  $V$  are stationary and only consider the energy cost for transmitting the data since it is much bigger than the energy cost for receiving [8].

For the coherence of terminology, we use *direction* to denote antenna sector uniformly in following discussion. Using such directional antennas model, only nodes located within one node's switched-on *directions* can receive the signal sent by the node. We adopt the following notations throughout the paper.

- $V$ : the set of network nodes.

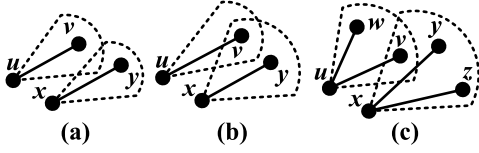


Fig. 1. The interfering relationship between directions

- $n$ : the number of network nodes, where  $n = |V|$ .
- $m$ : the number links in  $G$ , where  $|E| = m$ .
- $(s, D)$ : a broadcast/multicast request, where  $D \subseteq V \setminus \{s\}$  is the destination set, and when the request is broadcast,  $D = V \setminus \{s\}$ .
- $l(i)$ : the number of *directions* of a node  $v_i$  in  $V$ . Specially,  $l = \max_{1 \leq i \leq n} l(i)$  which is maximum number of *directions* for all nodes in  $V$ . In general,  $l \ll n$ .
- $d_{ij}$ : the  $j^{\text{th}}$  *direction* of node  $v_i$ ,  $1 \leq i \leq n, 1 \leq j \leq l(i)$ . Switching on a *direction* will lead to corresponding power consumption of a node.
- $w_{ij}$ : the transmission power consumption of node  $v_i$  switched on its  $j^{\text{th}}$  *direction*,  $1 \leq i \leq n, 1 \leq j \leq l(i)$ .
- $N^+(d_{ij})$ : the set of other nodes within transmission range of the  $j^{\text{th}}$  *direction* of node  $v_i$ . Since *directions* of a node maybe overlapping, a node may locate in two *directions* of one node.

When the network nodes are equipped with the directional antennas, the broadcasting/multicasting problems not only need to determine a set of relay nodes, but also need to select which *directions* of each node in the relay set should be switched on.

### B. Interference Model

As mentioned earlier, the goal of our algorithms is to conserve power consumption of the networks and reduce the inherent radio interference. To achieve this goal, our methods consider switching the antenna *directions* of each relay node, minimizing total power cost of the broadcast/multicast tree while minimizing total inherent interference of the tree. Now we are ready to introduce the interference model used in this paper. We assume that a node can only transmit or receive and be involved in a single communication session at one time. This interference model is an extension of the one defined in [15].

We firstly define interfering links based on *directions*. As shown in Fig. 1(b), if the *direction* corresponding to  $(x, y)$  and the *direction* corresponding to  $(u, v)$  cover a common node in  $(y, v)$ , we say that the two links interfere with each other. Because simultaneously transmit along both links will lead to collisions at the receiver. For the scenario that a direction can support multiple links, if two directions cover at least one common node, we consider that all links of one direction interfere with all links of other direction each other. Note that two directions may be identical node's sectors. As shown in Fig.1(c), links  $(x, y)$  and  $(x, z)$  interfere with links  $(u, v)$  and  $(u, w)$  each other. Although the links  $(u, w)$  and  $(x, z)$  in Fig.1(c) can exist at one time in some case, this

makes the link  $(u, v)$  become unusable for routing discover. Such potential interference may decrease the efficiency of a routing algorithm. So we consider they are interfering. By this definition, the two links in Fig.1(a) do not interfere with each other. From above the definition of interference, we can define the interference set of a *directions*. For each *direction*  $d_{ij}$ , we denote  $IE(d_{ij})$  to the set of links that interfere with one of links supported by  $d_{ij}$ .

In the following, we introduce the definition of interference we used in this paper. Let  $LD(v_i, v_k)$  represents the amount of traffic going through link  $(v_i, v_k)$ . In the initial network status, we assume all links in the network have one unit traffic load. The interference value of a *direction* is the sum of link load of the links, which interfere with one of links supported by this *direction*.

**Definition 1** (Direction Interference): Let  $d_{ij}$  be a node  $v_i$ 's  $j^{\text{th}}$  *direction*. The interference value of a *direction*  $d_{ij}$  (i.e. the  $j^{\text{th}}$  sector of node  $i$ 's directional antenna), denoted by  $f_{ij}$ , is the sum of the link loads among all links in  $IE(d_{ij})$ , i.e.,

$$f_{ij} = \sum_{e' \in IE(d_{ij})} LD(e'). \quad (1)$$

### C. Problem Formulations

In this subsection, we will formally define the problems to be studied.

For a broadcast/multicast request  $(s, D)$ , let  $T$  in  $G$  be a broadcast/multicast routing tree rooted at  $s$  and spanning all nodes in  $D$ . There are two kinds of nodes: the nodes (i.e. non-leaf nodes in  $T$ ) that need to transmit/relay messages and the nodes (i.e. leaf nodes in  $T$ ) that only receive message. We assume that a node costs energy only when it does transmissions. Let  $RT(T)$  denote the set of relay nodes. Let  $ON(i)$  denote IDs of  $v_i$ 's all switched-on *directions* in  $T$ . The total power cost of the broadcast/multicast routing tree  $T$  can be represented as:

$$P(T) = \sum_{v_i \in RT(T)} \sum_{j \in ON(i)} w_{ij}$$

The total interference of the broadcast/multicast routing tree  $T$  can be represented as:

$$L(T) = \sum_{v_i \in RT(T)} \sum_{j \in ON(i)} f_{ij}$$

For the delay-bounded scenario, we also give the definition of transmission delay of a broadcast/multicast routing tree. Assume  $d_T(s, v_i)$  is the length (by delay weight  $d$ ) of the path in  $T$  from  $s$  and a node  $v_i$ . The delay of a broadcast/multicast routing tree  $T$ , denoted by  $DL(T)$ , can be defined as  $\max\{d_T(s, v_i)\}$  for all node  $v_i$  in  $D$ . It is easy to see that  $DL(T) \geq DL(T_{SPT})$ , where  $T_{SPT}$  is the shortest path (by delay weight  $d$ ) tree rooted at  $s$  and spanning all nodes in  $D$ . Let  $P_{total}$  and  $I_{total}$  be a total power tolerance and a total interference tolerance respectively.  $DL$  is a user-specified threshold for transmission delay.

**Problem 1 (D-IPCB/IPCM):** The interference and power constrained broadcasting/multicasting with directional antennas problem can be formally represented as following: Given a broadcast/multicast request  $(s, D)$  and  $n$  nodes in network and each node  $v_i$  is equipped with a switched beam directional antenna, finding a broadcast/multicast routing tree  $T$  for  $(s, D)$  with minimum  $\max\{P(T)/P_{total}, L(T)/I_{total}\}$ .

**Problem 2 (DB-D-IPCB/IPCM):** The delay-bounded interference and power constrained broadcasting/multicasting with directional antennas problem can be formally represented as following: Given a broadcast/multicast request  $(s, D)$  and  $n$  nodes in network and each node  $v_i$  is equipped with a switched beam directional antenna, finding a broadcast/multicast routing tree  $T$  for  $(s, D)$  with minimum  $\max\{P(T)/P_{total}, L(T)/I_{total}\}$  and  $DL(T) \leq DL$ .

Note that the tolerance values  $P_{total}$  and  $I_{total}$  in above problems are the user-defined system parameters which can construct the comparable relative between  $P(T)/P_{total}$  and  $L(T)/I_{total}$ .  $P(T)/P_{total}$  and  $L(T)/I_{total}$  are the ratios of  $P(T)$  to  $P_{total}$  and  $L(T)$  to  $I_{total}$ . By carefully setting the values of  $P_{total}$  and  $I_{total}$ , routing trees obtained will not only be power-efficient, but also have very low interference values.

**Theorem 1:** The D-IPCB/IPCM and DB-D-IPCB/IPCM problems are NP-Hard.

**Proof:** It is easy to know that the D-MEB/MEM problems are special cases of the D-IPCB/IPCM problems. Since the D-MEB/MEM problems are NP-Hard [11] [13], therefore the D-IPCB/IPCM problems are NP-Hard. Clearly, DB-D-IPCB/IPCM problems are the special cases of the D-IPCB/IPCM problems, therefore, DB-D-IPCB/IPCM problems are also NP-Hard. ■

#### IV. THE CONSTRUCTION OF INTERFERENCE AND POWER CONSTRAINED BROADCAST/MULTICAST ROUTING TREE

In this section, we will propose an approximation algorithm and a heuristic for the D-IPCB/IPCM problem. Before introducing our algorithms, we study the edge weight-based multi-constrained directed Steiner tree problem along the lines of [21], [22], which can be used as a key subroutine to design the approximation algorithm for the D-IPCB/IPCM problem.

##### A. The Multi-Constrained Steiner Tree Problem

Given an edge-weighted directed graph  $H = (V, E, \vec{w})$ ,  $\vec{w} = (w_1, \dots, w_K)$  is a weight vector, and  $w_k(e) > 0$  is the  $k^{th}$  weight of edge  $e$ ,  $\forall e \in E$ ,  $1 \leq k \leq K$ . For a tree  $T$ , the  $k^{th}$  weight of  $T$  denoted by  $w_k(T)$ , is the sum of the  $k^{th}$  weights over the edges on  $T$ , i.e.  $w_k(T) = \sum_{e \in T} w_k(e)$ . Let  $C(T) = \max_{1 \leq k \leq K} \{w_k(T)/W_k\}$ . The *Multi-Constrained directed Steiner Tree* problem (MCST) can be presented as follow:

**Problem 3 (MCST):** Given an edge-weighted directed graph  $H = (V, E, \vec{w})$ , with  $K$  non-negative real-value edge weights  $w_k(e)$ ,  $1 \leq k \leq K$ , associated with each edge  $e \in E$ ; a constraint positive constant vector  $W = (W_1, \dots, W_K)$ ; a source node

$s$  and a destination set  $D \subseteq V \setminus \{s\}$ . Finding a directed Steiner Tree  $T$ , rooted at  $s$  and spanning all nodes in  $D$ , such that:  $C(T) = \max_{1 \leq k \leq K} \{w_k(T)/W_k\}$  is minimized.

The directed Steiner tree problem is NP-Hard [23], we believe that this problem is harder. We need to propose approximation algorithm for this problem.

##### 1) An Approximation Scheme for MCST

In order to solve the MCST problem, we transfer the MCST problem to the minimum directed Steiner tree problem. Based on  $H = (V, E, \vec{w})$ , we construct a new edge-weighted directed graph  $H' = (V, E, w_{max})$ , where the edge weight  $w_{max}(e) = \max_{1 \leq k \leq K} \{w_k(e)/W_k\}$  for each edge  $e \in E$ . And then apply an existing directed Steiner tree algorithm with respect to  $w_{max}(e)$  to solve the MCST problem. This algorithm can be formally presented as Algorithm 1.

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##### Algorithm 1 Algorithm for the MCST problem

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**Input:** A directed graph  $H = (V, E, \vec{w})$  with  $n$  node,  $m$  arcs and each arc with multi-weights  $w_k(e)$ ,  $1 \leq k \leq K$ . A positive constant constraint vector  $W = (W_1, \dots, W_K)$ . A source  $s$  and a node set  $D \subseteq V \setminus \{s\}$ .

**Output:** A directed Steiner tree  $T$  rooted at  $s$  and spanning all nodes in  $D$ .

**Begin:**

- 1) Initialization. Based on  $H$ , compute the auxiliary edge weight  $w_{max}(e)$  for each arc in  $E$  to construct the edge-weighted directed auxiliary graph  $H'$ .
- 2) Computation. With respect to the auxiliary edge weight  $w_{max}(e)$ , find an approximate, minimum edge-weighted directed Steiner tree  $T$  by using an existing algorithm.

**End.**

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##### 2) Theoretical Analysis

**Lemma 1:** Let  $T_{opt}$  is the optimal tree for MCST problem, the total edge weight (by  $w_{max}(e)$ ) of  $T_{opt}$  is at most  $K$  times of  $C(T_{opt})$ .

**Proof:** Since  $T_{opt}$  is an optimal solution to the MCST problem, for  $1 \leq k \leq K$ , we have:

$$\sum_{e \in T_{opt}} \frac{w_k(e)}{W_k} = \frac{w_k(T_{opt})}{W_k} \leq C(T_{opt}) \quad (2)$$

Therefore,

$$\sum_{e \in T_{opt}} \sum_{k=1}^K \frac{w_k(e)}{W_k} = \sum_{k=1}^K \frac{w_k(T_{opt})}{W_k} \leq K \cdot C(T_{opt}) \quad (3)$$

Notice that  $w_{max}(e) = \max_{1 \leq k \leq K} \{w_k(e)/W_k\}$ , we can have:

$$w_{max}(e) \leq \sum_{k=1}^K \frac{w_k(e)}{W_k} \quad (4)$$

By (2), (3) and (4), we know that:

$$\sum_{e \in T_{opt}} w_{max}(e) \leq K \cdot C(T_{opt}) \quad (5)$$

This lemma thus follows. ■

**Theorem 2:** If there is an approximation algorithm  $A$  for directed minimum Steiner tree problem with performance ratio  $Q$ , then using  $A$  in Algorithm 1, we can get an  $K \cdot Q$ -approximation algorithm for the MCST problem.

**Proof:** It is easy to know that Algorithm 1 can correctly get a feasible solution for MCST problem. We assume  $T_{opt}$  and  $T_a$  is optimal tree for MCST problem and the solution returned by Algorithm 1 respectively. We need to prove that:  $C(T_a) \leq K \cdot Q \cdot C(T_{opt})$ . Notice that  $T_a$  obtained by Algorithm 1 is the  $Q$ -approximate for minimum directed Steiner tree (SMT) problem by edge weight  $w_{max}(e)$ . Let  $SMT_{opt}$  be the optimal solution for directed minimum Steiner tree problem (by  $w_{max}(e)$ ) for  $s$  and  $D$ . We have that:

$$\sum_{e \in T_a} w_{max}(e) \leq Q \cdot \sum_{e \in SMT_{opt}} w_{max}(e) \quad (6)$$

and

$$\sum_{e \in SMT_{opt}} w_{max}(e) \leq \sum_{e \in T_{opt}} w_{max}(e) \quad (7)$$

In addition,

$$\begin{aligned} C(T_a) &= \max_{1 \leq k \leq K} \left\{ \frac{w_k(T_a)}{W_k} \right\} = \max_{1 \leq k \leq K} \left\{ \sum_{e \in T_a} \frac{w_k(e)}{W_k} \right\} \\ &\leq \sum_{e \in T_a} \max_{1 \leq k \leq K} \left\{ \frac{w_k(e)}{W_k} \right\} = \sum_{e \in T_a} w_{max}(e) \end{aligned} \quad (8)$$

Combining **Lemma 1** and formulas (6)-(8), we have:

$$\begin{aligned} C(T_a) &\leq \sum_{e \in T_a} w_{max}(e) \leq Q \cdot \sum_{e \in SMT_{opt}} w_{max}(e) \\ &\leq Q \cdot \sum_{e \in T_{opt}} w_{max}(e) \leq Q \cdot K \cdot C(T_{opt}) \end{aligned}$$

This proves the theorem. ■

Since the graph  $H$  has at most  $m$  arcs, construct the graph  $H'$  takes  $O(mK)$  time. For the minimum edge-weight directed Steiner tree problem, it is well known that there is  $O(|D|^\epsilon)$ -approximation algorithm for any  $0 < \epsilon \leq 1$  with time complexity  $O(n^{1/\epsilon} |D|^{2/\epsilon})$ , where  $n$  and  $|D|$  are the number of graph vertexes and Steiner terminals, respectively [23]. Therefore, we can apply this algorithm to get a solution for the MCST problem and the time complexity is  $O(mK + n^{1/\epsilon} |D|^{2/\epsilon})$  at the worst case.

### B. Algorithms for D-IPCB/IPCM Problem

As mentioned earlier, the D-IPCB/IPCM problem involves with not only the choice of the relay nodes, but also the switch of each relay node's *directions*. Based on the algorithm for MCST problem, we propose an approximation algorithm for the D-IPCB/IPCM problem.

Firstly, we construct a directed, edge-weight auxiliary graph  $G_A(N, E_A, c, f, d)$  to model the switch of each node's antenna *directions*, and then transform the original problem to the

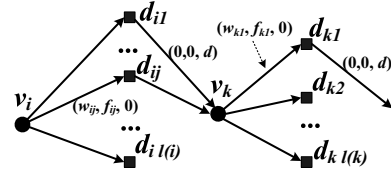


Fig. 2. The Construction of the auxiliary graph  $G_A$

edge weight-based problem. We construct the auxiliary graph  $G_A$  as follows.

For each node  $v_i$  in  $V$ , we also use vertex  $v_i$  in  $G_A$  to represent original network node  $v_i$  and then, vertices  $d_{ij}$ ,  $j = 1, 2, \dots, l(i)$  represent  $v_i$ 's *directions*. As shown in Fig. 2, directed edge  $(v_i, d_{ij})$  means that network node  $v_i$  switches on its direction  $j$  to transmit message. If  $v_k$  is located in the transmission range of  $v_i$ 's  $j^{th}$  direction, there is a directed edge  $(d_{ij}, v_k)$  from  $d_{ij}$  to  $v_k$ . i.e., if node  $v_i$  can transmit message to  $v_k$  by using its  $j^{th}$  direction, there is a directed edge  $(d_{ij}, v_k)$ . For the sake of convenience, vertex  $v_i$  is called as network vertex, vertex  $d_{ij}$  is called *direction* vertex and vertex  $d_{ij}$  is a *direction* vertex derived from  $v_i$ .

We get an edge-weighted directed graph  $G_A = (N, E_A, w, l)$ , where  $N = V \cup \{d_{ij} | v_i \in V, 1 \leq j \leq l(i)\}$ ,  $E_A = \{(v_i, d_{ij}) | 1 \leq i \leq n, 1 \leq j \leq l(i)\} \cup \{(d_{ij}, v_k) | v_k \text{ is located in } v_i \text{'s } j^{th} \text{ direction}\}$ . The edge weight functions  $c$ ,  $f$  and  $d$  represent the power cost, interference value and transmission delay, respectively. For each arc from a network vertex to a *direction* vertex, we assign  $c((v_i, d_{ij})) = w_{ij}$ ,  $f((v_i, d_{ij})) = f_{ij}$  and  $d((v_i, d_{ij})) = 0$ . For each arc from a *direction* vertex to a network vertex, we assign  $c((d_{ij}, v_k)) = 0$ ,  $f((d_{ij}, v_k)) = 0$  and  $d((d_{ij}, v_k))$  is set to a delay of message transmitted by using direction  $d_{ij}$  from  $v_i$  to  $v_k$ , which is relate to the wireless communication technology. For each *direction* vertex, it has only one in-arc which is from its network node. The auxiliary graph has at most  $((l+1)n)$  vertices and at most  $(nl+m)$  edges, where  $l = \max\{l(i), i = 1, 2, \dots, n\}$ . The graph transformation can be accomplished in  $O(ln^2)$ . Having the edge-weighted, directed auxiliary graph  $G_A$ , for any broadcast or multicast request  $(s, D)$ , we still denote  $(s, D)$  as multicast request in  $G_A$ .

**Theorem 3:** The D-IPCB/IPCM problem can be equivalently transformed to the 2-constrained directed minimum Steiner tree problem in  $G_A$ .

**Proof:** It is easy to know that we must prove that the following two cases are right:

(1) for any broadcast/multicast routing tree  $T$  in  $G$  for D-IPCB/IPCM problem, there is a 2-constrained Steiner tree (MCST)  $T'$  in  $G_A$  corresponding to  $T$ , and Vice versa.

(2)  $T_{opt}$  is a optimal routing tree in  $G$  rooted at the source  $s$  and spanning all nodes in  $D$  for D-IPCB/IPCM problem,  $T_{opt}$  is a minimum 2-constrained Steiner tree for  $(s, D)$  in corresponding graph  $G_A$ , then  $C(T_{opt}) = C(T_{opt}')$ , where  $C(T_{opt}) = \max\{P(T_{opt})/P_{total}, L(T_{opt})/I_{total}\}$  for D-IPCB/IPCM problem in  $G$ .

We know (1) is obvious. In the following, we will prove (2).

Firstly, we prove that  $C(T^{opt}) \leq C(T_{opt})$ . Suppose that  $T_{opt}$  is a optimal routing tree in  $G$  rooted at the source  $s$  and spanning all nodes in  $D$  for D-IPCB/IPCM problem. Note that  $T_{opt}$  is a directed tree. Following the construction of  $G_A$ , there is a corresponding edge-weighted directed Steiner tree  $T$  in  $G_A$  rooted at  $s$  and spanning the vertices in  $D$ , and the cost of  $T$   $C(T)$  is equal to  $\max\{P(T_{opt})/P_{total}, L(T_{opt})/I_{total}\}$ . Let  $T^{opt}$  be a minimum 2-constrained directed Steiner tree in  $G_A$  for  $(s, D)$ , then  $C(T^{opt}) \leq C(T) = C(T_{opt})$ .

Secondly, we prove that  $C(T^{opt}) \geq C(T_{opt})$ . We have the following facts: Given a edge-weighted directed Steiner tree  $T$  in  $G$  rooted at  $s$  and spanning all vertices in  $D$ , then we can get a corresponding broadcast/multicast tree in ad hoc networks for the broadcast/multicast request  $(s, D)$ .

Suppose  $T^{opt}$  is a minimum 2-constrained directed Steiner tree for  $(s, D)$  in  $G_A$ . Then we can get a corresponding broadcast/multicast tree  $T'$  in  $G$  for D-IPCB/IPCM problem, in this case,  $C(T_{opt}) \leq C(T') = C(T^{opt})$ . The theorem follows. ■

From Theorem 3, the D-IPCB/IPCM problem in  $G$  can be equivalently transformed to the 2-constrained directed Steiner tree problem in  $G_A$ . We thus propose an approximation algorithm for the D-IPCB/IPCM problem by applying Algorithm 1 as a key subroutine and it can be described as follows (Algorithm 2):

---

**Algorithm 2** Algorithm for D-IPCB/IPCM problem

**Input:**  $n$  nodes in network and each node  $v_i$  is equipped with a switched beam directional antenna. A broadcasting/multicasting request  $(s, D)$ , the total interference tolerance  $I_{total}$  and total power tolerance  $P_{total}$ .

**Output:** A broadcast/multicast routing tree  $T$  for  $(s, D)$ .

**Begin:**

- 1: Initially, construct the directed auxiliary graph  $G_A = (N, E_A, c, f, d)$ .
- 2: Find a 2-constrained directed Steiner tree  $T'$  in  $G_A$  for  $(s, D)$  by using the Algorithm 1.
- 3: Obtain the broadcast/multicast  $T$  and switch on the *directions* for each non-leaf network node (i.e. transmit/relay node) to transmit data by using the information provided by  $T'$

**End.**

---

If we use an  $|D|^\epsilon$ -approximation algorithm for the minimum directed Steiner tree problem in [23] in Algorithm 2, From Theorem 2 and 3, we can get  $O(2 \cdot |D|^\epsilon)$ -approximate solution for the D-IPCB/IPCM problem.

**Theorem 4:** Algorithm 2 can correctly product a  $O(2 \cdot |D|^\epsilon)$ -approximate solution for the D-IPCB/IPCM problem in time  $O(n^{2l} + (nl)^{1/\epsilon} \cdot |D|^{2/\epsilon})$ .

In addition, we also propose a MST-based heuristic with lower time complexity. The main idea is that: find a directed MST rooted at  $s$  by adding arc by arc with minimum incremental value of  $\max\{P(T)/P_{total}, L(T)/I_{total}\}$ , and then prune the redundant arcs to achieve the directed Steiner tree. The time complexity of this algorithm is  $O((nl)^2)$ , which is

dominated by the computation of a directed MST. We study the performance of this heuristic by simulations and formally present this algorithm as Algorithm 3.

---

**Algorithm 3** Algorithm for D-IPCB/IPCM problem

**Input:**  $n$  nodes and each node  $v_i$  is equipped a switched beam directional antenna. A broadcasting/multicasting request  $(s, D)$ , the total interference tolerance  $I_{total}$  and total power tolerance  $P_{total}$ .

**Output:** A broadcast/multicast routing tree  $T$  for  $(s, D)$ .

**Begin:**

- 1: Initially, construct the directed auxiliary graph  $G_A = (N, E_A, c, f, d)$ . Let  $T' = \{s\}$  and  $U = N \setminus \{s\}$ .
- 2: **while**  $U \neq \emptyset$  **do**
- 3: Find a arc  $(u, v)$  from  $T'$  to  $U$ , which results in minimum incremental value of  $\max\{P(T')/P_{total}, L(T')/I_{total}\}$ .
- 4:  $T' = T' \cup \{v\}$ .
- 5:  $U = U \setminus \{v\}$ .
- 6: **end while**
- 7: Prune the redundant arcs to construct a directed Steiner tree  $T'$  rooted at  $s$  and spanning all nodes in  $D$ .
- 8: Obtain the broadcast/multicast  $T$  and switch on the *directions* for each non-leaf network node to transmit data by using the information provided by  $T'$ .

**End.**

---

V. THE CONSTRUCTION OF INTERFERENCE AND POWER CONSTRAINED BROADCAST/MULTICAST ROUTING TREE WITH BOUNDED TRANSMISSION DELAY

The transmission delay refers to the time for messages to travel across the network. In many applications such as sensor network, a sink needs to quickly disseminate user queries to the sensor nodes, the data by sensors also needs to be reported back to user as soon as possible. In such application, small transmission delays are highly expected. In this section, we will study the DB-D-IPCB/IPCM problem. Before introducing our algorithm, we study the delay-bounded multi-constrained Steiner tree problem, which can be used as a key subroutine to design the approximation algorithm for the DB-D-IPCB/IPCM problem. Additionally, a heuristic with lower time complexity also be presented in subsection V-B.

A. The Delay-Bounded Multi-Constrained Steiner Tree Problem

Given an directed graph  $H = (V, E, \vec{w})$  with edge weight vector  $\vec{w} = (w_1, \dots, w_K, d)$ ,  $d(e) > 0$  is the delay value and  $w_k(e) > 0$  is the  $k^{th}$  weight of edge  $e$ ,  $\forall e \in E, 1 \leq k \leq K$ . For a tree  $T$ , the  $k^{th}$  weight of  $T$  denoted by  $w_k(T)$ , is the sum of the  $k^{th}$  weights over the edges on  $T$ :  $w_k(T) = \sum_{e \in T} w_k(e)$ .  $d_T(r, u)$  is the transmission delay from  $T$ 's root  $r$  to a node  $u$  in  $T$  and be equal to sum of  $d(e)$  in the path from  $r$  to  $u$  in  $T$ . The Transmission Delay of a tree  $T$  is denoted as  $d(T)$  and equal to  $\max_{u \in T} d_T(r, u)$ . Let  $C(T) = \max_{1 \leq k \leq K} \{w_k(T)/W_k\}$  and  $w_{max}(e)$  be

equal to  $\max_{1 \leq k \leq K} \{w_k(e)/W_k\}$ . The *Delay-Bounded Multi-Constrained directed Steiner Tree* problem can be presented as follow:

**Problem 4 (DB-MCST):** Given an edge-weighted directed graph  $H = (V, E, \vec{w})$ , with  $K + 1$  non-negative real-value edge weights  $d(e), w_k(e), 1 \leq k \leq K$ , associated with each edge  $e \in E$ ; a positive constant *Transmission Delay* constraint  $DL$  and a positive constant constraint vector  $W = (W_1, \dots, W_K)$ ; a source node  $s$  and a destination set  $D \subseteq V \setminus \{s\}$ . Finding a directed Steiner Tree  $T$  for  $(s, D)$  such that:  $DL(T) \leq DL$  and  $C(T)$  is minimized.

1) *An Approximation Scheme for DB-MCST:*

Similar with subsection 4.1, we construct a new edge-weight directed graph  $H' = (V, E, w_{max})$  based on  $H$ , where  $w_{max}(e) = \max_{1 \leq k \leq K} \{w_k(e)/W_k\}$ , and set same delay value in  $H$  to each edge  $e \in E$ . Then we propose an algorithm to construct a delay-bounded directed Steiner tree  $T$  on  $H'$ . Initially,  $T$  contains only  $s$ . At each iteration, apply an existing delay-bounded shortest path algorithm to find a shortest path  $P$  (by  $w_{max}(e)$ ) from  $T$  to a node  $u$  in  $D$  such that  $d_P(s, u) \leq DL$ , and then merge  $P$  and  $T$  into a new tree  $T$ . Repeat this process until  $T$  spans all nodes in  $D$ .

For the delay-bounded shortest path problem, Xue *et al.* [24] proposed an  $(1 + \varepsilon)$ -approximation algorithm with time  $O(mn \log \log \log n + mn/\varepsilon)$ . Given a delay constraint  $DL$ , an approximation parameter  $\varepsilon > 0$ , and a pair of source-destination nodes, the algorithm in [24] find a source-destination path whose delay is at most  $DL$  and whose cost is no more than  $(1 + \varepsilon)$  times the cost of the optimal delay-constrained shortest path. We can use it in our algorithm. All logarithms are base-2 logarithms in this section.

The approximation algorithm for the DB-MCST problem can be formally described as follows.

2) *Theoretical Analysis*

**Theorem 5:** Algorithm 4 can correctly product an  $(1 + \varepsilon)K \cdot |D|$ -approximation solution for the delay-bounded multi-constrained directed Steiner tree (DB-MCST) problem with time  $O(mK + |D|(mn \log \log \log n + mn/\varepsilon))$ .

**Proof:** It is easy to know that Algorithm 4 can correctly get a feasible solution for the problem.

Let  $T_{opt}$  and  $T_a$  are optimal SMT for the DB-MCST problem and the solution returned by Algorithm 4 respectively. We need to prove that:  $C(T_a) \leq (1 + \varepsilon)K \cdot |D| \cdot C(T_{opt})$ .

Similar with the proof of **Lemma 1**, we also can prove that:

$$\sum_{e \in T_{opt}} w_{max}(e) \leq K \cdot C(T_{opt}). \quad (9)$$

Let  $P_1, P_2, \dots, P_r$  be the paths obtained at  $r = |D|$  iterative steps of Algorithm 4 respectively, and  $T_a = P_1 \cup P_2 \cup \dots \cup P_r$ . Note that we set  $w_{max}(e) = 0$  for each  $e \in P$  in line 5 of Algorithm 4, which means that find a shortest path to a destination node from  $T$ . It is easy to know that:

$$\sum_{e \in T_a} w_{max}(e) \leq \sum_{i=1}^r \sum_{e \in P_i} w_{max}(e). \quad (10)$$

---

#### Algorithm 4 Algorithm for the DB-MCST problem

---

**Input:** A directed graph  $H = (V, E, \vec{w})$  with  $n$  node,  $m$  arcs and each arc with multi-weights  $d(e), w_k(e), 1 \leq k \leq K$ , a transmission delay constraint  $DL$  and a positive constant constraint vector  $W = (W_1, \dots, W_K)$ . A source  $s$  and a node set  $D \subseteq V \setminus \{s\}$ .

**Output:** A directed Steiner tree  $T$  for  $(s, D)$ .

**Begin:**

- 1: Initialization. Based on  $H$ , compute  $w_{max}(e)$  for each arc in  $E$  to construct the edge-weighted directed auxiliary graph  $H'$ .
- 2: Let  $T = \{s\}$  and  $U = D$ .
- 3: **while**  $U \neq \emptyset$  **do**
- 4: Apply the algorithm in [24] to find a delay-bounded shortest path  $P$  in  $H'$  from  $s$  to a destination node  $u$  in  $U$  such that  $d_P(r, u) \leq DL$ .
- 5: Set the  $w_{max}(e) = 0$  for each  $e$  in  $P$ .
- 6: Merge  $T$  and  $P$  into a new tree  $T$ .
- 7:  $U = U \setminus \{u\}$ .
- 8: **end while**

**End.**

---

Let  $P_1^{opt}, P_2^{opt}, \dots, P_r^{opt}$  be the corresponding optimal shortest paths with respect to the auxiliary edge weight  $w_{max}(e)$ . Since the algorithm in [24] is a subroutine of Algorithm 4 which is a  $(1 + \varepsilon)$ -approximation for delay-bounded shortest path problem. Then, for  $1 \leq i \leq r$ , we have:

$$\sum_{e \in P_i} w_{max}(e) \leq (1 + \varepsilon) \cdot \sum_{e \in P_i^{opt}} w_{max}(e). \quad (11)$$

Since  $P_1^{opt}, P_2^{opt}, \dots, P_r^{opt}$  be the optimal shortest paths connecting the corresponding source and destinations with respect to the auxiliary edge weight  $w_{max}(e)$ . So the length (by  $w_{max}(e)$ ) of connecting the same source and destination is at least  $\sum_{e \in P_i^{opt}} w_{max}(e), 1 \leq i \leq r$ .

For  $1 \leq i \leq r$ , we have:

$$\sum_{e \in P_i^{opt}} w_{max}(e) \leq \sum_{e \in T_{opt}} w_{max}(e) \quad (12)$$

and

$$\sum_{i=1}^r \sum_{e \in P_i^{opt}} w_{max}(e) \leq r \cdot \sum_{e \in T_{opt}} w_{max}(e) \quad (13)$$

Combining formulas (10)-(13), we have:

$$\sum_{i=1}^r \sum_{e \in P_i} w_{max}(e) \leq r \cdot (1 + \varepsilon) \cdot \sum_{e \in T_{opt}} w_{max}(e) \quad (14)$$

and,

$$\sum_{e \in T_a} w_{max}(e) \leq r \cdot (1 + \varepsilon) \cdot \sum_{e \in T_{opt}} w_{max}(e) \quad (15)$$

Recall that  $w_{max}(e) = \max_{1 \leq k \leq K} \{w_k(e)/W_k\}$ , this implies:

$$C(T_a) \leq \sum_{e \in T_a} w_{max}(e) \quad (16)$$

Combining formulas (9), (15) and (16) and  $r = |D|$ , we have:

$$C(T_a) \leq \sum_{e \in T_a} w_{max}(e) \leq (1 + \varepsilon) \cdot |D| \cdot \sum_{e \in T_{opt}} w_{max}(e) \\ \leq (1 + \varepsilon) \cdot K \cdot |D| \cdot C(T_{opt}).$$

Therefore, the approximation ratio of Algorithm 4 is  $(1 + \varepsilon) \cdot K \cdot |D|$ .

Since there are  $m$  arcs in the directed graph  $H$ , the running time of the Step 1 takes time  $O(mK)$ . The while-loop has at most  $|D|$  iterations. For each loop, compute the delay-bounded shortest path takes time  $O(mn \log \log \log n + mn/\varepsilon)$ . The whole algorithm thus ends in time  $O(mK + |D|(mn \log \log \log n + mn/\varepsilon))$  at the worst case.

This theorem thus follows. ■

### B. Algorithms for DB-D-IPCB/IPCM Problem

Along the way proposed in subsection IV-B, we will solve the DB-D-IPCB/IPCM problem by transforming the original problem to the edge-weighted problem on auxiliary  $G_A$  and using the approximation scheme we proposed for the DB-MCST to design the approximation algorithm for the DB-D-IPCB/IPCM Problem. This approximation algorithm can be formally described as Algorithm 5.

---

#### Algorithm 5 Algorithm for DB-D-IPCB/IPCM problem

**Input:**  $n$  nodes in network and each node  $v_i$  is equipped with a *switched beam* directional antenna. A broadcasting/multicasting request  $(s, D)$ , a delay constraint  $DL$  and the total interference and power tolerances  $I_{total}$  and  $P_{total}$ .

**Output:** A broadcast/multicast routing tree  $T$  for  $(s, D)$  with  $DL(T) \leq DL$ .

**Begin:**

- 1: Initially, construct the directed auxiliary graph  $G_A = (N, E_A, c, f, d)$ .
- 2: Find an approximate, delay-bounded 2-constrained directed Steiner tree  $T'$  in  $G_A$  for  $(s, D)$ , using the Algorithm 4.
- 3: Obtain the broadcast/multicast  $T$  and switch on the *directions* for each non-leaf network node (i.e. transmit/relay node) to transmit data by using the information provided by  $T'$

**End.**

---

**Theorem 6** Algorithm 5 can correctly compute an  $O(2(1 + \varepsilon) \cdot |D|)$ -approximate solution for the DB-D-IPCB/IPCM problem in time  $O(n^3 l |D| (\log \log \log(nl) + 1/\varepsilon))$ .

**Proof:** Since the DB-D-IPCB/IPCM problem can be equivalently transformed to the delay-bounded 2-constrained directed Steiner tree problem in  $G_A$ . The correctness of Algorithm 4 guarantees that Algorithm 5 can correctly product an  $O(2(1 + \varepsilon) \cdot |D|)$ -approximate solution for the DB-D-IPCB/IPCM problem.

Construct  $G_A$  takes  $O(ln^2)$  time. Step 3 takes time  $O(nl)$ . When the original network has  $n$  nodes and  $m$  links, the

auxiliary graph has at most  $nl + n$  vertexes and  $nl + m$  arcs in  $G_A$ . The time complexity of Algorithm 5 is dominated by the running time of Algorithm 4. Since  $n \gg l$  and  $O(m) \leq O(n^2)$ , the time complexity of Algorithm 5 is:  $O(n^3 l |D| (\log \log \log(nl) + 1/\varepsilon))$ . If  $\varepsilon$  is set to 1, we can get a  $4 \cdot |D|$ -approximate solution for the DB-D-IPCB/IPCM problem in time  $O(n^3 l |D| \log \log \log(nl))$  at the worst case. This proves the theorem. ■

Because Algorithm 5 has a relatively high time complexity, in the following, we propose a heuristic with lower time complexity for the DB-D-IPCB/IPCM problem. This heuristic algorithm is formally presented as Algorithm 6.

---

#### Algorithm 6 Algorithm for DB-D-IPCB/IPCM problem

**Input:**  $n$  nodes in network and each node  $v_i$  is equipped with a *switched beam* directional antenna. A broadcasting/multicasting request  $(s, D)$ , a delay constraint  $DL$  and the total interference and power tolerances  $I_{total}$  and  $P_{total}$ .

**Output:** A broadcast/multicast routing tree  $T$  for  $(s, D)$  with  $DL(T) \leq DL$ .

**Begin:**

- 1: Initially, construct the directed auxiliary graph  $G_A = (N, E_A, c, f, d)$ .
- 2: Construct a directed MST  $T'$  rooted at  $s$  by adding arc by arc with minimum incremental value of  $\max\{P(T')/P_{total}, L(T')/L_{total}\}$  and then prune the redundant arcs.
- 3: Apply a Depth-First Search (DFS) on  $T'$  as follows: traverse an arc-by-arc walk from  $s$  through the nodes in  $T'$ . If node  $u$  in  $D$  is visited the first time and the delay constraint is not met, delete the arc to  $u$  in  $T'$  and the shortest path (by delay weight  $d$ ) from  $s$  to  $u$  is added into  $T'$ .
- 4: Obtain the broadcast/multicast  $T$  and switch on the *directions* for each non-leaf network node (i.e. transmit/relay node) to transmit data by using the information provided by  $T'$

**End.**

---

It is easy to know that Algorithm 6 can correctly deliver a feasible solution for the DB-D-IPCB/IPCM problem with time  $O((nl)^2)$ .

The centralized algorithms presented in subsection IV-B and V-B all can be implemented in distributed manner by adding few assumptions. Due to the page limitation, we leave the descriptions of these distributed implementations in the extension of this paper.

## VI. PERFORMANCE EVALUATIONS

In this section, we evaluate the performance of our algorithms by simulations. Since the D-IPCB/IPCM (DB-D-IPCB/IPCM) problem considers the tradeoff between power and interference and our work is the first result for the problem, we compare our algorithms with the directed Steiner tree algorithm [23] ( $\varepsilon = 1/2$ ) based on the power weight



and interference weight of arcs, respectively. The simulate parameters and some notations can be summarized as follows:

- $n$ : the number of network nodes (be set to 101) which be randomly placed in a  $1000 \times 1000$   $m^2$  2-D space.
- $DK$ : the mean value of the number of *directions* which is set to 6.
- $R$ : the mean value of all *directions'* transmission radius, which is set to 200 m. The power model of each *direction* is  $P = C \cdot \theta \cdot r^\alpha / 360$ , where the constant  $C$  and  $\alpha$  are set to 0.0001 and 2 respectively in all simulations.  $\theta$  is beam-width of a *direction*.
- $d$ : the delay for 1-hop communication.  $d$  is set to 1 [25].  $DL$  is a user-specified threshold for the transmission delay of a broadcast/multicast routing tree.
- $s$  and  $D$ : the source node and destination set.  $D \subseteq V \setminus \{s\}$  and  $|D|$  range from 20 to 100.
- $C(T)$ : the cost of tree  $T$  ( $= \max\{P(T)/P_{total}, L(T)/I_{total}\}$ ).
- $ATI$ : Average Total Interference.
- $ATPC$ : Average Total Power Cost.
- AA2: the Approximation Algorithm 2 for D-IPCB/IPCM in section IV-B. The directed minimum Steiner tree algorithm in [23] ( $\varepsilon = 1/2$ ) is applied in Step 2 of Algorithm 1, which is a key subroutine of Algorithm 2.
- MST-B: the MST-based heuristic Algorithm 3 in subsection IV-B for D-IPCB/IPCM.
- AP: the Algorithm in [23] ( $\varepsilon = 1/2$ ) with respect to the Power weight of arcs of  $G_A$ , whose total power cost is denoted to  $P$  for each network instance.
- AI: the Algorithm in [23] ( $\varepsilon = 1/2$ ) with respect to the Interference weight of arcs of  $G_A$ , whose total interference is denoted to  $I$  for each network instance.
- $I_{total}$  and  $P_{total}$ : the total interference and total power tolerances for our algorithms (are equal to  $I \cdot K_1$  and  $P \cdot K_2$  respectively, where  $K_1$  and  $K_2$  are the *Bound Ratio*).

We present averages of 100 separate runs for each result shown in the following figures. In each run of the simulations, for each network instance, we randomly select  $s$  and  $D$ . Network instance  $G$  got by switching on all *directions* of each node, which is not connected from source node to all other nodes, is discarded. The instance  $G$ , which does not meet  $d_G(s, u) \leq DL$  for all node  $u$  in  $D$ , is also discarded. Then we run our algorithms on the networks.

In Fig. 3, we compare the total power and total interference by all algorithms with  $|D|$  varying. We fixed  $n$ ,  $R$ ,  $DK$  while vary  $|D|$  and the *bound ratio*  $(K_1, K_2)$ . As we can see, in all subfigures, the average total power cost and total interference value increase with the increase of the  $|D|$  for all algorithms. The AP algorithm only considers the power weight of each arc, so the average total power cost of the trees computed by AP are the least. Similarly, AI only considers the weight of interference, so it gets the trees with the least average total interference. However, the algorithm only based on the power weight (interference) of each arc may lead to increment

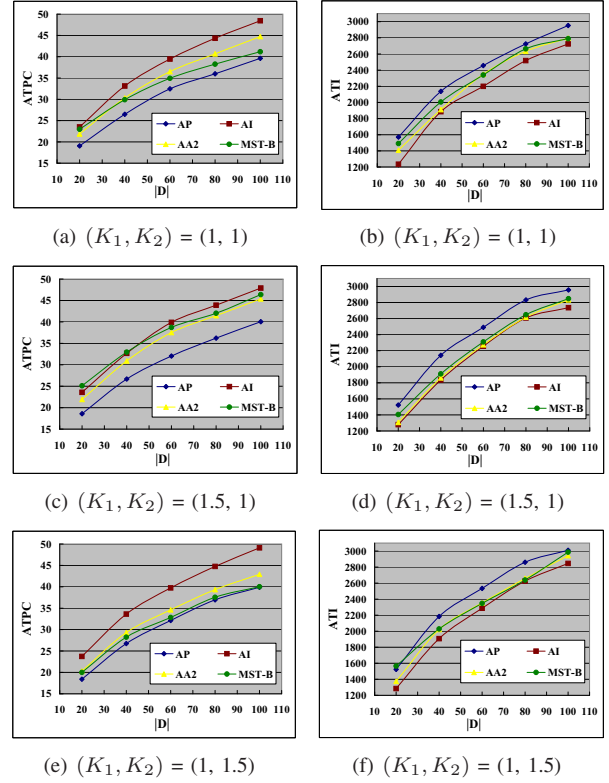
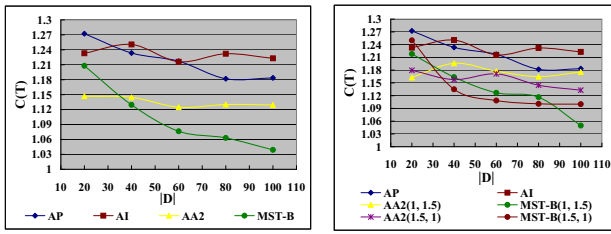


Fig. 3. Average Total Power Cost and Average Total Interference with different  $|D|$  and Bound Ratio

of total interference value (total power cost) of the trees, respectively. From all subfigures in Fig.3, AI (or AP) have the maximum average total power cost (total interference), and our algorithms can find a trade-off between the total power cost and total interference value. From Fig. 3(a) and (b) (setting  $(K_1, K_2)=(1, 1)$ ) we can see that our algorithms dramatically reduce the total power and total interference compared with the AI and AP algorithms, respectively. It is because, the optimization objective of our algorithms is minimizing  $C(T)=\max\{P(T)/(P \cdot K_1), L(T)/(I \cdot K_2)\}$ , some arcs which can result in much increment of  $C(T)$  compared with other arcs have to be excluded by our algorithms. Hence, this property makes the arcs selected by our algorithm have balanced weights between power and interference.

Comparing Fig. 3(a) with 3(c) and Fig. 3(b) with 3(d), it is showed that the total power increases and interference decreases as  $K_1$  increases when  $K_2$  does not change, respectively. Comparing Fig. 3(a) with 3(e) and Fig. 3(b) with 3(f), it is showed that the total power decreases and interference increases as  $K_2$  increases when  $K_1$  does not change, respectively. These results are consistent to characters of our problems. These due to the fact that a relatively greater bound ratio  $K_1$  makes our algorithms mainly consider the weight of interference. For example, about AA2, if  $K_1 = 1.5 \leq K_2 = 1$ ,  $w_{max}(e)=\max\{w_{ij}/P \cdot K_1, f_{ij}/I \cdot K_2\}$  has higher opportunity to be computed to  $f_{ij}/I \cdot K_2$ . Then the tree constructed by AA2 approaches to the minimum interference tree (computed by



(a)  $(K_1, K_2) = (1, 1)$ ,  $C(T)$  vs.  $|D|$  (b)  $C(T)$  vs.  $|D|$

Fig. 4. The tree cost  $C(T)$  with different  $|D|$

AI). Note that the total interference value and total power cost never exceed the bounds gave by AI and AP (In our simulation, the AA2, AI and AP are all based on the same directed Steiner tree algorithm). Therefore, our algorithms can be used to find a controllable trade-off between the total tree power cost and total tree interference value. Some anomalistic points of these curves in Fig. 3 are acceptable since the networks are different among every running when the  $|D|$  is varying.

In Fig.4, we compare the value of solutions for the D-IPCB/IPCM by all algorithms with  $|D|$  varying. We also can see that the broadcast/multicast routing trees computed by our algorithms achieve balanced value between total power cost and total interference value. Furthermore, the  $C(T)$  of the trees constructed by our algorithms are dramatically smaller than the trees computed by AP and AI. Small value of  $C(T)$  means that the tree  $T$  not only be energy efficient, but also has low interference. With the increment of  $|D|$ , the heuristic MST-B performs better than the approximation algorithm AA2.

We also conducted simulations for the delay-bounded scenarios, and we can achieve similar observations and conclusions with the previous scenarios when  $|D|$  and the *bound ratio* are varied.

## VII. CONCLUSIONS

We have studied the interference and power constrained broadcast/multicast routing problems with directional antennas in wireless ad hoc networks. We proposed several approximation algorithms and heuristics for the D-IPCB/IPCM and DB-D-IPCB/IPCM problems, and our algorithm can provide a controllable trade-off between total power cost and total interference. Specially, we also investigate the multi-constrained directed Steiner tree problem and the delay-bound multi-constrained directed Steiner tree problem, and proposed approximation algorithms for these two problem, respectively. To our best knowledge, this is the first result for the (delay-bounded) multi-constrained directed minimum Steiner tree problem with theoretical performance guarantee.

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