Probabilistic Analysis for Mixed Criticality Scheduling with SMC and AMC

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***Abstract*—This paper introduces probabilistic analysis for fixed priority preemptive scheduling of mixed criticality systems on a uniprocessor using the Adaptive Mixed Criticality (AMC) and Static Mixed Criticality (SMC) schemes. We compare this analysis to the equivalent deterministic methods, highlighting the performance gains that can be obtained by utilising more detailed information about worst-case execution time estimates described in terms of probability distributions.**

I. Introduction

In 2007, Vestal [13] introduced a mixed criticality task model, where each task is represented by multiple worst-case execution time (WCET) estimates that are determined at different levels of assurance. For example, the WCET estimates $C(LO)$ and $C(HI)$ are the estimates for low-assurance and high-assurance respectively in a dual-criticality system. By contrast, in this paper we use a richer model based on probabilistic worst-case execution time (pWCET) distributions, provided by probabilistic analysis of worst-case execution times of tasks [8], [1]. Figure 1 illustrates this richer model, where the line on the graph denotes the 1-CDF (Complementary Cumulative Distribution Function or *exceedance function*) of the pWCET for a task. The 1-CDF describes the probability that the WCET of the task exceeds any particular value given on the x-axis. The high-assurance estimate $C(HI)$ may be obtained using an exceedance probability (on the y-axis) of $p(HI) = 10^{-9}$, where 10^{-9} denotes an acceptable threshold on the failure probability for each job of the task at the high-assurance level[1]. Thus in a mixed criticality system with a probabilistic description, the scheduler can safely ignore any execution demand beyond $C(HI)$ because its probability of occurrence is below the threshold required. Further, at a lower level of assurance (say with acceptable threshold $p(LO) = 10^{-5}$), the scheduler can ignore any execution demand beyond $C(LO)$ because its probability of occurrence is below the threshold for that level of assurance.

In this paper, we focus on the development of a probabilistic mixed criticality framework under the Fixed-Priority Preemptive Scheduling (FPPS) policy. Our choice of FPPS is motivated by the fact that it is a standard scheduling policy employed in many applications. There are two well-known FPPS schemes and corresponding deterministic schedulability analyses in the literature on mixed critiality scheduling; Static Mixed Criticality (SMC) [3] and Adaptive Mixed Criticality (AMC) [2]. We develop probabilistic analyses for both of these schemes. (A brief description of these schemes is given in Section II).

[1]We note that the relationship between failure rates and acceptable thresholds on failure probabilities is not direct, but rather depends on various factors considered in fault tree analysis [9]. In this paper, we assume that suitable values for the thresholds are given, and make a somewhat arbitrary choice of values for illustration purposes

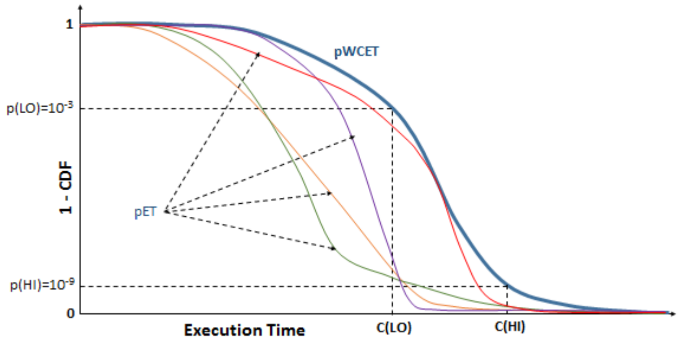


Fig. 1: Exceedance function or 1-CDF for the pWCET distribution of a task, and also pET distributions for different jobs of the same task.

A. Related work

A growing number of studies have been introduced for mixed criticality real-time scheduling, an overview of these approaches can be found in the review on mixed criticality systems [6]. Previous work on probabilistic analysis and mixed criticality systems was done for the Earliest Deadline First (EDF) scheduling policy. Santinelli and George [12] presented preliminary work that investigates the probabilistic C-space, showing how schedulability varies with task execution times under EDF. Guo et al. [10] extended the mixed criticality task model with a single exceedance probability value for the low assurance budget of each HI-criticality task, and used probabilistic analysis to improve schedulability. In contrast, we focus on the FPPS policy, and consider a richer task model with pWCET distributions.

II. System Model

In this paper, we are interested in the Fixed Priority Preemptive Scheduling of a Mixed Criticality System (MCS) comprising a static set of n sporadic tasks which execute on a single processor. We assume without loss of generality that each task τ_i has a unique priority. We further assume a discrete time model in which all task parameters are given using integers.

Each task τ_i is defined by its period (or minimum arrival interval), relative deadline, worst-case execution time, and level of criticality (defined by the system engineer responsible for the entire system): (T_i, D_i, C_i, L_i). We restrict our attention to constrained-deadline systems in which $D_i \leq T_i$ for all tasks.

In a mixed criticality system, further information is needed in order to perform schedulability analysis. Thus a task τ_i may be defined by: (T_i, D_i, $\vec{C_i}$, L_i), where $\vec{C_i}$ is a vector of values – one per criticality level, with the constraint $L1 > L2 \Rightarrow C_i(L1) \geq C_i(L2)$ for any two criticality levels $L1$ and $L2$. In this paper we are concerned with dual criticality systems, with criticality levels LO and HI (where LO < HI).

Using a *deterministic* representation each LO-criticality task τ_i has a single worst-case execution time estimate $C_i(LO)$, while each HI-criticality task τ_i has two worst-case execution time estimates $C_i(LO)$ and $C_i(HI)$ with $C_i(HI) \geq C_i(LO)$. We use $p(LO)$ (and $p(HI)$) to denote the worst-case probability that $C_i(LO)$ (resp. $C_i(HI)$) is exceeded during the execution of any single job of the task. Thus $p(LO)$ and $p(HI)$ reflect the confidence that we have in the two WCET estimates. Note, in this paper, we assume that these probabilities are the same for all tasks and so drop the index. We use exemplar values for $p(LO)$ and $p(HI)$ of 10^{-5} and 10^{-9} respectively. (Note for ease of presentation, we also drop the index for $C(LO)$ and $C(HI)$ when using these terms in a generic way; nevertheless, these values are specific to each task).

By contrast, using a *probabilistic* representation, each task τ_k (of LO- or HI-criticality) has a probabilistic worst-case execution time distribution (pWCET) $\mathcal{C}_k$. Further, there is a correspondence between the probabilistic and deterministic representations. Considering the 1-CDF (exceedance function), $f(\mathcal{C}_k)$ for task τ_k, $C_k(LO)$ (resp. $C_k(HI)$) in the deterministic representation corresponds to the value of $f(\mathcal{C}_k)$ at a probability of exceedance of $p(LO)$ (resp. $p(HI)$). Note beyond this correspondence, we make no assumptions in our analysis about the form of the distribution. We assume that the WCET values of two tasks C_i and C_j are independent and so too are their pWCET distributions $\mathcal{C}_i$ and $\mathcal{C}_j$ [7].

For simplicity of presentation, we assume that $C(HI)$ is not exceeded and that $p(HI)$ therefore represents the probability that the WCET of a HI-criticality task is $C(HI)$ (rather than exceeds $C(HI)$).

The SMC and AMC scheduling schemes investigated in this paper both use budget enforcement by the Real-Time Operating System (RTOS) to ensure that LO-criticality tasks cannot execute for more than their LO-criticality execution time budget $C(LO)$. With the AMC scheme, the RTOS also uses the $C(LO)$ budget for each HI-criticality task to determine if a mode change should take place. The system moves from LO- to HI-criticality mode if this budget is reached without the job completing. Once HI-criticality mode is entered, then with AMC, any jobs of LO-criticality tasks that have already started can continue to execute; however, no further jobs of LO-criticality tasks can be released. (This simple extension to the original AMC scheme, which called for jobs of LO-criticality tasks to be aborted on entering HI-criticality mode, is permitted by the analysis [2] and was proposed in [4] as a means of avoiding difficulties in dealing with aborted jobs). With SMC, jobs of LO-criticality tasks continue to be released in HI-criticality mode. Full details of both schemes can be found in [2] and [4].

In any processor busy period, where all jobs of HI-criticality tasks complete without exceeding their LO-criticality budgets, the system is said to be in *LO-criticality mode*; otherwise it is said to be in *HI-criticality mode*. (We assume that at an idle instant when there are no jobs with outstanding execution, the system may revert back to LO-criticality mode). There are different requirements on schedulability that apply in the different modes of the system.

We use R_i to refer to the *deterministic* worst-case response time (WCRT) of task τ_i, and $\mathcal{R}_i$ to refer to the probabilistic worst-case response time (pWCRT) distribution which may be computed using pWCET values.

In LO-criticality mode, jobs of LO-criticality tasks must have a worst-case Deadline Miss Probability (DMP) that is no greater than a specified threshold $H(LO)$ (for example 10^{-5}). We assume these thresholds are the same value for all LO-criticality tasks and so drop the index. Using deterministic analysis, this requirement may be satisfied by showing that the tasks are schedulable i.e. have a worst-case response time $R_i(LO) \leq D_i$, computed using execution times of $C(LO)$. Using the probabilistic analysis developed in this paper, the requirement may be satisfied directly by determining the worst-case deadline miss probability. This is achieved by computing a probabilistic worst-case response time distribution (pWCRT) and determining the value of the 1-CDF (Cumulative Distribution Function) at a response time corresponding to the task's deadline D_i, thus computing the probability of missing the deadline.

Jobs of HI-criticality tasks must have a worst-case deadline miss probability that is no greater than a specified threshold $H(HI)$ (for example 10^{-9}). This requirement applies to all modes, and may be met via deterministic methods or via calculating the appropriate probabilistic worst-case response time distribution and comparing it with the task's deadline. In both cases, due account needs to be taken of interference from LO-criticality tasks. In addition, we note that jobs of LO-criticality tasks may fail to meet their timing requirements by not completing by their execution time budget $C(LO)$. The probability of this happening depends upon the budget set and may be read from the pWCET distribution for the task. It is the same irrespective of whether we use deterministic or probabilistic analysis. (The high assurance RTOS is assumed to enforce this budget at runtime).

Finally we note a subtle but important point about using pWCET distributions and probabilistic analysis, which does not occur with deterministic analysis. This point rests on the fact that when we analyse LO-criticality behaviour, we can use low assurance information e.g. $C(LO)$ WCET estimates, pWCET distributions for LO-criticality tasks etc. However, when we analyse HI-criticality behaviour, we must be sure to use only high assurance information e.g. pWCET distributions for HI-criticality tasks, and rely on the high assurance RTOS to enforce $C(LO)$ budgets for LO-criticality tasks. We return to this point in Section IV-A.

III. Recap of Existing Probabilistic Schedulability Analysis

In this section, we recapitulate the probabilistic response time analysis for FPPS originally derived in [11] for tasks with both worst-case execution times and minimum inter-arrival times described by random variables. Since in this paper the minimum inter-arrival time for each task is a constant, we present only a simplified version of this analysis. First we recap the basic terminology and operators used.

We distinguish between *full* distributions and *partial* distributions. A *full* distribution $\mathcal{Z}$ has probabilities which sum to 1. Such a distribution may be split into two (or more) *partial* distributions $\mathcal{X}$ and $\mathcal{Y}$ such that $\forall v$ $P(\mathcal{Z} = v) = P(\mathcal{X} = v) + P(\mathcal{Y} = v)$. We say that

$\mathcal{Z} = \mathcal{X} \oplus \mathcal{Y}$ where $\oplus$ is the *coalescence* of the two distributions via the addition of the probabilities for each value. In contrast, the sum $\mathcal{Z}$ of two *independent* distributions $\mathcal{X}$ and $\mathcal{Y}$ is given by their *convolution* $\mathcal{X} \otimes \mathcal{Y}$ where $P\{\mathcal{Z} = z\} = \sum_{k=-\infty}^{k=+\infty} P\{\mathcal{X} = k\}P\{\mathcal{Y} = z - k\}$.

In [11], Maxim and Cucu-Grosjean showed that considering all valid patterns of job releases, the worst-case response time distribution of a job of task τ_i occurs for the first job of τ_i released simultaneously with jobs of all higher priority tasks, which are then re-released as soon as possible. We can therefore compute an upper bound on the pWCRT distribution $\mathcal{R}_i$ of task τ_i as follows.

The worst-case response time distribution for task τ_i is first initialized to:

$$\mathcal{R}_i^0 = \mathcal{B}_i \otimes \mathcal{C}_i \tag{1}$$

where the backlog $\mathcal{B}_i$ at the release of τ_i is given by:

$$\mathcal{B}_i = \bigotimes_{j \in hp(i)} \mathcal{C}_j \tag{2}$$

The worst-case response time is then updated iteratively for each preemption as follows:

$$\mathcal{R}_i^m = (\mathcal{R}_i^{m-1,head} \oplus (\mathcal{R}_i^{m-1,tail} \otimes \mathcal{C}_k^{pr})) \tag{3}$$

Here, m is the index of the iteration. $\mathcal{R}_i^{m-1,head}$ is the part of the distribution $\mathcal{R}_i^{m-1}$ that is not affected by the preemption under consideration (i.e. it only contains values $\leq t_m$ where t_m is the time of the preemption). $\mathcal{R}_i^{m-1,tail}$ is the remaining part of the distribution $\mathcal{R}_i^{m-1}$ that may be affected by the preemption. Finally, $\mathcal{C}_k^{pr}$ is the pWCET distribution of the preempting task τ_k.

Iteration ends when there are no releases left from jobs of higher priority tasks at time instants smaller than the largest value in the response time distribution currently obtained. Iteration may also be terminated once any new preemptions are beyond the deadline of the task. Once iteration is complete, the worst-case Deadline Miss Probability valid for any job of task τ_i is given by:

$$DMP_i = P(\mathcal{R}_i > D_i). \tag{4}$$

IV. Probabilistic Mixed Criticality Schedulability Analysis

In this section, we introduce probabilistic response time analysis for the SMC and AMC scheduling schemes, referred to as pSMC and pAMC analysis respectively.

A. pWCET Distributions Used

In our analysis, we make use of different types of pWCET distribution which we now describe. We use the term *partLO* to denote a partial distribution $\mathcal{X}$ formed from the full pWCET distribution $\mathcal{Z}$ of a task by taking only those values that represent completion of the task in no more than its budgeted LO-criticality execution time $C(LO)$. Thus $\mathcal{X}$ is such that $\forall v \leq C(LO)\ P(\mathcal{X} = v) = P(\mathcal{Z} = v)$ and $\forall v > C(LO)\ P(\mathcal{X} = v) = 0$.

As discussed in Section II, we assume that the value $C(HI)$ is the final and therefore largest value in the pWCET distribution for a HI-criticality task. Figure 2a illustrates both $full$ and $partLO$ distributions for a HI-criticality task. (Note the shape of these distributions is for illustration purposes only. No specific distribution is assumed by the analysis).

In the case of a LO-criticality task, we are also interested in the $partLO$ distribution, since this describes the behaviour expected of it by the system designer. In addition, with both the SMC and AMC schemes, if a job of such a task executes for its execution time budget $C(LO)$ without signaling completion, then the job will be aborted as a result of budget enforcement by the RTOS. We need to also take this behaviour into account when computing the interference on other tasks of lower priority. Thus we form a further partial distribution for each LO-criticality task denoted by *partBE* (meaning budget exceeded). This distribution $\mathcal{Y}$ has a single value of $C(LO)$ with a probability which equates to the sum of the probabilities for all larger values from the full distribution, i.e. $P(\mathcal{Y} = C(LO)) = P(\mathcal{Z} > C(LO))$. Note that $\mathcal{X} \oplus \mathcal{Y}$ forms the full distribution that we use for a LO-criticality task. This is illustrated in Figure 2b. We refer to this distribution as *fullBE* (the full distribution accounting for budget exceedance).

When we analyse the HI-criticality mode (e.g. computing the pWCRT distribution for a HI-criticality task), then we cannot trust the full pWCET distributions obtained for LO-criticality tasks. The reason for this is that LO-criticality tasks are not developed to the same rigourous standards as HI-criticality tasks. Thus, in the analysis we provide for the HI-criticality mode, we must not make any assumptions about their correct behaviour. Instead, we assume the worst; that such tasks may enter an infinite loop, and be aborted at their $C(LO)$ budget by the RTOS, which is itself a trusted component developed to the standards required for HI-criticality operation. When we are analysing HI-criticality mode, we therefore assume that LO-criticality tasks have a *degen* (meaning full degenerate) distribution which has a single value $C(LO)$ with probability 1 (see Figure 2c).

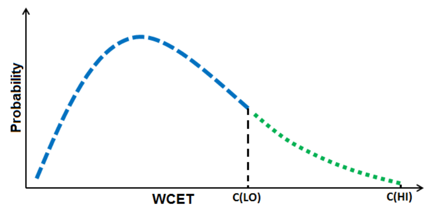
pWCRT distributions are composed from pWCET distributions using the analysis described in Section III. When partial pWCET distributions are used (for example representing LO-criticality mode only), then the resultant pWCRT distribution is also a partial distribution, giving probabilities that are conditional on being in that mode.

When analysing the Deadline Miss Probability for LO-criticality tasks, we use the *partLO* distributions, since we are only interested in the probability that tasks exceed their deadlines *and* the system remains in LO-criticality mode, i.e. no $C(LO)$ budgets are exceed. The component of a full pWCRT distribution that is lost represents only those scenarios where the system enters HI-criticality mode, and hence where there is no requirement for LO-criticality tasks to meet their deadlines.

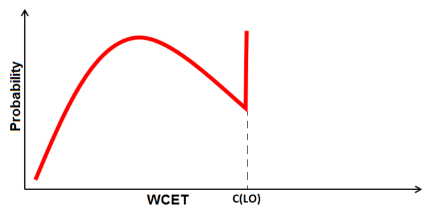
We note that a partial distribution $\mathcal{Y}$ describing the behaviour in HI-criticality mode can be obtained from a full distribution $\mathcal{Z}$ describing the behaviour in all modes, by *subtracting* the partial distribution $\mathcal{X}$ describing the behaviour in LO-criticality mode. By subtraction we mean the subtraction of probabilities for the corresponding values, $\forall v\ P(\mathcal{Y} = v) = P(\mathcal{Z} = v) - P(\mathcal{X} = v)$, i.e. the counterpart of coalescence. We write this as $\mathcal{Y} = \mathcal{Z} \ominus \mathcal{X}$. Here, $\mathcal{Y}$ is well defined provided that $\mathcal{X}$ is a partial distribution of $\mathcal{Z}$. This is the case in all uses of the $\ominus$ operator in this paper, since $\mathcal{Z}$ is composed from full distributions and $\mathcal{X}$ from corresponding partial distributions for the same tasks.

As a building block for the analysis in the following sections, we use a function $\text{pRTA}(\tau_k, \delta, \delta LO, \delta HI, t)$ defined as follows.

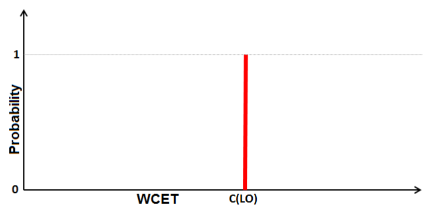
Definition 1: $\text{pRTA}(\tau_k, \delta, \delta LO, \delta HI, t)$ is a function which returns a (full or partial) pWCRT distribution for task τ_k computed using the analysis described in Section III. This analysis starts from an initial distribution of type δ for task τ_k, and uses pWCET distributions of types δLO and δHI



(a) *partLO* and *full* probability distribution for a HI-criticality task.



(b) *fullBE* distribution for a LO-criticality task.



(c) *degen* distribution for a LO-criticality task.

Fig. 2: Illustrations of the full and partial distributions used in probabilistic analysis.

for preempting (i.e. higher priority) LO-criticality and HI-criticality tasks respectively. Further, jobs of higher priority LO-criticality tasks are only included in the computation of the pWCRT distribution if their release times are no later than time t. The parameters δ, δLO, and δHI may take values *partLo*, *fullBE*, *degen*, and *full* when the function is used.

B. *pSMC Analysis*

Recall that with the SMC scheme, LO-criticality tasks may execute in HI-criticality mode. The only constraint on their execution being budget enforcement. We note that for a LO-criticality task, there are three possibilities:

(i) The task overruns its budget, the probability for this case is given by the $partBE$ distribution for the task;
(ii) The task does not overrun its budget and the system operates in LO-criticality mode;
(iii) The task does not overrun its budget and the system operates in HI-criticality mode.

The partial pWCRT distributions for cases (ii) and (iii) are derived below. For a HI-criticality task, we assume that there can be no overrun of the $C(HI)$ budget, and thus $C(HI)$ is effectively the maximum value in the pWCET distribution. Thus there are just two cases to consider, equivalent to (ii) and (iii) above. The partial pWCRT distributions for those cases are also derived below.

1) LO-criticality mode: We determine schedulability in LO-criticality mode using the *partLO* distribution for the task of interest τ_k, and the $fullBE$ distribution for higher priority, LO-criticality tasks, since they may overrun their budgets, but there is no behaviour of these tasks that can cause HI-criticality mode to be entered. Finally, we need only consider the $partLO$ distribution for HI-criticality tasks, as the remaining part of the full distribution for these tasks implies that the system enters HI-criticality mode, hence:

$$\mathcal{R}_k(LO) = \text{pRTA}(\tau_k, partLO, fullBE, partLO, \infty) \quad (5)$$

Equation (5) applies to both LO- and HI-criticality tasks. In each case it provides the partial pWCRT distribution for task τ_k conditional on the system operating in LO-criticality mode and the task not exceeding its own budget.

2) HI-criticality mode: When analysing a HI-criticality task τ_k executing in HI-criticality mode, we can no longer trust the behaviour of LO-criticality tasks. Thus we must use degenerate distributions equating to the execution time budget $C(LO)$ for higher priority, LO-criticality tasks.

First, we compute the pWCRT distribution for both modes. Here, we begin with the full distribution for HI-criticality task τ_k and similarly include the full distribution for preempting higher priority, HI-criticality tasks.

$$\mathcal{Z}_k = \text{pRTA}(\tau_k, full, degen, full, \infty) \quad (6)$$

Then we determine the partial pWCRT distribution for LO-criticality mode:

$$\mathcal{X}_k = \text{pRTA}(\tau_k, partLO, degen, partLO, \infty) \quad (7)$$

Finally, we obtain the partial pWCRT distribution for HI-criticality mode as a simple subtraction:

$$\mathcal{R}_k(HI) = \mathcal{Z}_k \ominus \mathcal{X}_k \quad (8)$$

Equation (8) upper bounds the pWCRT distribution for a HI-criticality task τ_k conditional on the system operating in HI-criticality mode.

C. *pAMC Analysis*

The AMC and SMC schemes have identical behaviour in LO-criticality mode, hence the analysis given in Section IV-B also provides pAMC analysis for both LO- and HI-criticality tasks in LO-criticality mode. That leaves pAMC analysis of HI-criticality tasks in HI-criticality mode.

Again, we first compute the pWCRT distribution for both HI- and LO-criticality modes. Here, we begin with the full distribution for task τ_k and similarly include the full distribution for preempting higher priority, HI-criticality tasks. As this analysis is for HI-criticality mode, we assume that the behaviour of LO-criticality tasks cannot be trusted and therefore make use of the *degen* distributions for those tasks.

$$\mathcal{Z}_k = \text{pRTA}(\tau_k, full, degen, full, R(LO)) \quad (9)$$

Then we determine the partial pWCRT distribution for LO-criticality mode:

$$\mathcal{X}_k = \text{pRTA}(\tau_k, partLO, degen, partLO, R(LO)) \quad (10)$$

Note, in (10) for symmetry and obvious compatibility with (9) we limit the jobs of higher priority LO-criticality tasks to those released by time $R(LO)$ given by the deterministic analysis for AMC [2]; however, since $R(LO)$ is effectively computed using the maximum values in the $degen$ distributions for LO-criticality tasks and $partLO$ distributions for HI-criticality tasks, then in practice this does not restrict the distribution $\mathcal{X}_k$ in any way. Note that if $R(LO)$ cannot be obtained by deterministic analysis i.e. the task is unschedulable and $R(LO) > T$ then $R(LO)$ may be assumed to be infinite and the pAMC analysis reduces to the pSMC analysis. Finally, we obtain the partial pWCRT distribution for HI-criticality mode as a simple subtraction:

$$\mathcal{R}_k(HI) = \mathcal{Z}_k \ominus \mathcal{X}_k \quad (11)$$

Equation (11) upper bounds the pWCRT distribution for a HI-criticality task τ_k conditional on the system operating in HI-criticality mode. Together with (5), this completes the analysis for $pAMC$.

Comparing (9) and (10) with (6) and (7), it is easy to see that the pAMC analysis dominates pSMC. The only difference is the discounting of LO-criticality preemptions after time $R(LO)$ in the case of pAMC. Thus all task sets that are deemed schedulable[2] by pSMC analysis are also schedulable according to the pAMC analysis. Further, we note that the pAMC analysis dominates deterministic analysis for AMC, and similarly, pSMC analysis dominates deterministic analysis for SMC. This can be seen by considering the distributions used in the probabilistic analyses. These distributions and the resulting pWCRT distributions satisfy the *limit condition* [11]. The maximum values in each input distribution are the same as the values used in the corresponding deterministic analysis, thus the maximum value in the output pWCRT distributions are the same as the deterministic WCRTs.

D. *Probabilistic schedulability*

For a LO-criticality task to meet its schedulability requirements, then its execution time must not exceed its $C(LO)$ budget with more than a specified probability. This is guaranteed, as with deterministic analysis, by setting its execution time budget no lower than $C(LO)$. Secondly, its pWCRT distribution conditional on not exceeding its budget and that the system remains in LO-criticality mode must give a worst-case Deadline Miss Probability that does not exceed the specified threshold, i.e. $P(\mathcal{R}_k(LO) > D_k) \leq H(LO)$. This can be determined for both pSMC and pAMC using (5).

For a HI-criticality task to meet its schedulability requirements, then execution of no more than $C(HI)$ is assumed to be assured. Further, its pWCRT distribution (valid for both modes combined) must give a worst-case Deadline Miss Probability that does not exceed the specified threshold i.e. $P(\mathcal{R}_k > D_k) \leq H(HI)$, where $\mathcal{R}_k = \mathcal{R}_k(LO) \oplus \mathcal{R}_k(HI)$. This can be determined for pSMC using (5) and (8), and for pAMC using (5) and (11).

V. Evaluation

In this section, we present an empirical investigation, examining the effectiveness of the probabilistic analysis techniques (pSMC and pAMC) introduced in this paper, in particular examining the performance improvements that can be obtained with respect to the corresponding deterministic analyses.

A. *Task set parameter generation*

The task set parameters used in our experiment were randomly generated as follows:

- Task utilisations ($U_i = C_i/T_i$) (for LO-criticality mode) were generated using the UUnifast algorithm [5], giving an unbiased distribution of values.
- Task periods were generated according to a log-uniform distribution from $10ms$ to $1000ms$.
- The LO-criticality execution time of each task was set based on the utilisation and period: $C_i(LO) = U_iT_i$.
- The HI-criticality execution time of each task was a fixed multiplier of the LO-criticality execution time, $C_i(HI) = CF \cdot C_i(LO)$ (default $CF = 1.5$), where CF is called the Criticality Factor.
- The probability that a generated task was a HI-criticality task was given by the parameter CP (default $CP = 0.5$).
- Task deadlines were constrained, chosen from a uniform distribution in the range $[CF.C(LO), T]$.

[2]Recall that by schedulable we mean that LO-criticality tasks must not exceed their DMP threshold in LO-criticality mode, and HI-criticality tasks must not exceed their DMP threshold in either mode.

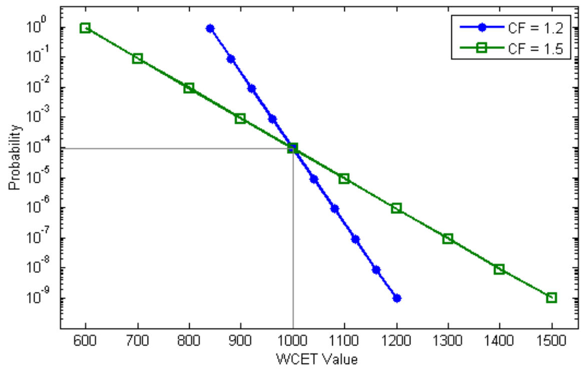


Fig. 3: Example of possible pWCET distributions.

- Task priorities were set in deadline monotonic order.

We generated the pWCET distribution for each task via extrapolation from the $C(LO)$ and $C(HI)$ parameter values. We assumed that the probability of exceeding $C(LO)$ is $p(LO) = 10^{-5}$, and that the probability of executing for, but not exceeding $C(HI)$ is $p(HI) = 10^{-9}$. To determine intermediate points, we assumed that the pWCET distributions have an exponential tail (as is the case with a Gumbel distribution). Thus we assumed that the 1-CDF of the pWCET was a straight line on an exceedance graph with probabilities given on a log scale, as depicted in Figure 3.

The Thresholds $H(LO)$ and $H(HI)$ on the maximum acceptable deadline miss probabilities for LO- and HI-criticality tasks were set to 10^{-5} and 10^{-9} respectively.

B. *Schedulability tests*

We investigated the performance of the following techniques and associated schedulability tests.

- pSMC: Probabilistic SMC analysis (Section IV-B).
- pAMC: Probabilistic AMC analysis (Section IV-C).
- dSMC: Deterministic SMC analysis [2].
- dAMC: Deterministic AMC analysis [2].
- dUB: Task sets pass this ‘test’ if they are schedulable according to deterministic analysis of FPPS in each of the individual LO-criticality and HI-criticality modes with priorities in deadline monotonic order. This is a necessary test for any fixed priority preemptive mixed criticality scheduling algorithm [2].
- pUB: Task sets pass this ‘test’ if they are schedulable according to probabilistic analysis of FPPS in each of the individual LO-criticality and HI-criticality modes with priorities in deadline monotonic order.

The dominance relationships between the algorithms and tests implies a strict ordering to the lines on the graph: pUB dominates pAMC which dominates pSMC. Similarly dUB dominates dAMC which dominates dSMC. Further pUB dominates dUB, pAMC dominates dAMC, and pSMC dominates dSMC. The purpose of the experiment is to examine the relative performance of the different schemes.

In our experiment, the LO-criticality utilisation was varied from 0.05 to 1. For each utilisation value, 1000 task sets were generated and the schedulability of those task sets determined for the different schemes.

Figure 4 plots the percentage of task sets generated that were deemed schedulable for a system of 10 tasks, with on average 50% of those tasks having HI-criticality (CP = 0.5) and each task having a HI-criticality execution time that is 1.5 times its LO-criticality execution time (CF = 1.5). The utilisation values (x-axis) are computed using the $C(LO)$ values and periods for each task.

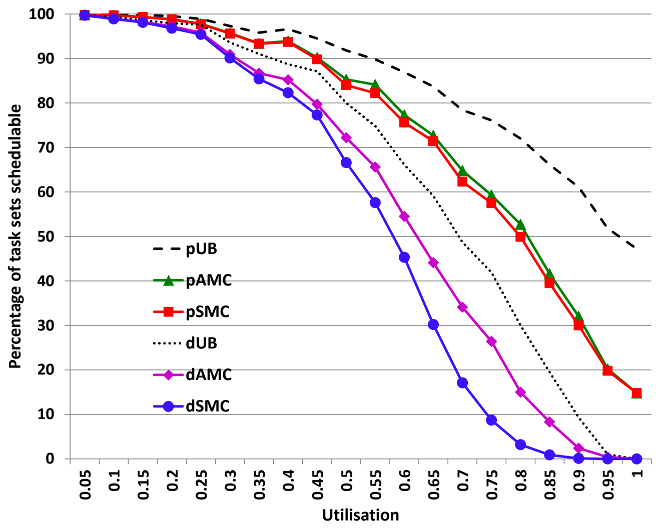


Fig. 4: Percentage of task sets deemed schedulable according to the four tests: dSMC, dAMC, pSMC and pAMC.

Figure 4 shows that the probabilistic analysis (pAMC and pSMC) provides substantially improved performance compared to deterministic analysis (dAMC and dSMC), with many more task sets deemed schedulable. This is because the probabilistic analysis is able to account for the full extent of the pWCET distributions, and thus the very small probability that multiple jobs take long execution times leading to a very long response time. We note that pAMC and pSMC may deem some tasks sets with LO-criticality utilisation greater than 1 schedulable. This is correct, and is a reflection of the shape of the pWCET distributions (see Figure 3). Recall that $C(LO)$ has a probability of exceedance of 10^{-5} thus once the distributions for a number of tasks are convolved, the probability that all of them execute for $C(LO)$ or more becomes very small. Further, we note that the results for pSMC and pAMC converge at a very high values of LO-criticality utilisation. This is because finite values for $R(LO)$ can no longer be computed via deterministic analysis. We note that using the deterministic $R(LO)$ value in pAMC leads to only a small improvement over pSMC. In future, we will investigate whether an improved pAMC analysis can be obtained by making use of the probabilistic $\mathcal{R}(LO)$ distribution.

VI. Conclusions

In this paper we introduced probabilistic analysis for fixed priority preemptive scheduling of mixed criticality systems under the SMC [3] and AMC schemes [2]. This analysis makes use of probabilistic worst-case execution time (pWCET) distributions to compute probabilistic worst-case response time distributions (pWCRT) and thus the worst-case deadline miss probability (DMP) for each task under SMC and AMC. Provided that the DMP is below the appropriate threshold[3] for that task, then it is declared schedulable.

The main contributions of this paper are as follows:

- Introducing probabilistic analysis of mixed criticality systems based on a richer mixed criticality model using pWCET distributions.
- Deriving probabilistic worst-case response time analysis for both LO- and HI-criticality tasks under the SMC and AMC schemes.
- Showing via an experimental evaluation that the probabilistic analyses, pSMC and pAMC, substantially out-perform their deterministic counterparts. This is due to the fact that low-probability unfavorable events such as all HI-criticality jobs simultaneously requiring additional execution time can be soundly ignored.

[3]Derived from acceptable failure rates for tasks of that criticality.

In future, we intend to explore whether the probabilistic analysis framework can provide improved support for LO-criticality tasks based on the observation that the LO-criticality WCET estimates used in the deterministic representation are in reality tunable execution time budgets at different levels of assurance. We expect that the more effective probabilistic analysis will enable these budgets to be increased for both LO- and HI-criticality tasks, thus reducing the chance that HI-criticality mode is entered and lowering the likelihood that LO-criticality jobs have to be dropped or aborted.

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